INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES
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International eJournals
International Journal of Mathematical Sciences, Technology and Humanities 3 (2011) 25 – 36

RADIATION AND FIRST ORDER CHEMICAL REACTION EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE

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ABSTRACT:

An exact analysis of thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past a exponentially accelerted infinite isothermal vertical plate, in the presence homogeneous chemical reaction of first order has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane. The effects of velocity, temperature and concentration are studied for different physical parameters like thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing values of the chemical reaction parameter or radiation parameter. But the trend is just reversed with respect to parameter 'a' and time t.

Key words: first order, chemical reaction, thermal radiation, isothermal,vertical plate.

Mathematics Subject Classification: 76R10

Introduction:

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [1] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al* [2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al* [3]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery[5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[6]. The governing equations were solved analytically. Das *et al*[4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Gupta[7] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[8] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar[9]. Basant Kumar Jha *et al* [10] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

However the study of thermal radiation effects on unsteady flow past a exponentially accelerated isothermal vertical plate in the presence of chemical reaction of first order is not studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past a expontially accelerated isothermal vertical plate, in the presence of thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

1. Basic Equations and Analysis

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

$$\begin{array}{llll} t' \le 0: & u = 0, & T = T_\infty, & C' = C'_\infty \quad for\ all \quad y \\ t' > 0: & u = u_0 \exp(a't'), & T = T_w, & C' = C'_w \quad at \quad y = 0 \\ & u = 0, & T \to T_\infty, & C' \to C'_\infty \quad as \quad y \to \infty \end{array} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\,\sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p\, \frac{\partial T}{\partial t'} = k\,\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3\,(T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{array}{l} U = \dfrac{u}{u_0}, t = \dfrac{t' u_0^2}{\nu}, Y = \dfrac{y u_0}{\nu}, \theta = \dfrac{T - T_\infty}{T_w - T_\infty}, \\ Gr = \dfrac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \dfrac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \dfrac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3}, \end{array} \tag{8}$$

$$R = \frac{16a^{*}\nu^{2}\sigma T_{\infty}^{3}}{ku_0^2},\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ K = \frac{\nu K_l}{u_0^2}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0, & \text{for all} & Y, t \le 0 \\ t > 0: & U = \exp(at), & \theta = 1, & C = 1, & \text{at} & Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & \text{as} & Y \to \infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \tag{13}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{14}$$

$$\begin{aligned} U = \frac{\exp(at)}{2}&\left[\exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at})\right] \\ &- e\exp(bt)\left[\exp(2\eta\sqrt{bt})\, erfc(\eta + \sqrt{bt}) + \exp(-2\eta\sqrt{bt})\, erfc(\eta - \sqrt{bt})\right] \\ &+ (e + f)\, erfc(\eta) \\ &- f\exp(ct)\left[\exp(2\eta\sqrt{ct})\, erfc(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})\, erfc(\eta - \sqrt{ct})\right] \\ &- e\left[\exp\left(2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\mathrm{Pr}} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\mathrm{Pr}} - \sqrt{at}\right)\right] \end{aligned}$$

$$+ e\exp(bt)\left[\exp\left(-2\eta\sqrt{\Pr(b+c)t}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{(b+c)t}\right)\right.$$

$$\left.+\exp\left(2\eta\sqrt{\Pr(b+c)t}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{(b+c)t}\right)\right]$$

$$-f\left[\exp\left(2\eta\sqrt{K\,t\,Sc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{K\,t}\right)+\exp\left(-2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{K\,t}\right)\right]$$

$$+f\exp(dt)\left[\exp\left(-2\eta\sqrt{Sc(K+d)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(K+d)t}\right)\right.$$

$$\left.+\exp\left(2\eta\sqrt{Sc(K+d)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(K+d)t}\right)\right] \quad (15)$$

where, $b=\dfrac{R}{Pr}, c=\dfrac{R}{1-Pr}, d=\dfrac{K\,Sc}{1-Sc}, e=\dfrac{Gr}{2c(1-Pr)}$, $f=\dfrac{Gc}{2d(1-Sc)}$ and $\eta=Y/2\sqrt{t}$

3 Discussion of Results

The numerical values of the velocity, temperature andc concentartion are computed for different physical parameters like thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air($Pr=0.71$). The purpose of the calculations given here is to assess the effects of the parameters R, K, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The concentartion profiles for different values of the Schmidt number $(Sc=0.16, 0.3, 0.6, 2.01), K=0.2$ at time $t=0.2$ are shown in figure 1. The effect of Schmidt number is important in concentration field. Figure 2 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter (K=0.2, 2, 5, 10) at $t=0.2$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the velocity increases with decreasing chemical reaction parameter or Schmidt number.

Figure 3 illustrates the effect of the velocity for different values of the chemical reaction parameter ($K=0.2, 7, 15), R=10, Gr=Gc=5, R=0.2$ and $t=0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter. The effect of velocity for different values of the radiation parameter ($R=0.2, 5, 20), K=0.2, Gr=Gc=5$ and $t=0.4$ are shown in figure 4. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation. The velocity profiles for different time ($t=0.2, 0.3, 0.4), R=0.2,$ $Gr=Gc=2$ and $K=0.2$ are shown in Figure 5. This shows that the velocity increases gradually with respect to time t. The velocity profiles for different (a = 0.2 ,0.5 ,0.9) Gr=Gc=5 and R=0.2 at $t=0.2$ are studied and presented in figure 6. It is observed that the velocity increases with increasing values of a.

The temperature profiles for different values of thermal radiation parameter($R = 2,5,7,10$), in the presence of air at time $t = 0.4$ are shown in figure 7. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

4. Conclusion

An exact analysis of thermal radiation effects on unsteady flow past a exponentially accelerated infinite isothermal vertical plate, in the presence of chemical reaction of first order has been studied. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like R, K, Gr, Gc, Sc and t are studied. The study concludes that the velocity increases with decreasing chemical reaction parameter K or thermal radiation parameter R. The trend is just reversed with respect to time t. As expected, the plate concentration increases with decreasing chemical reaction parameter.

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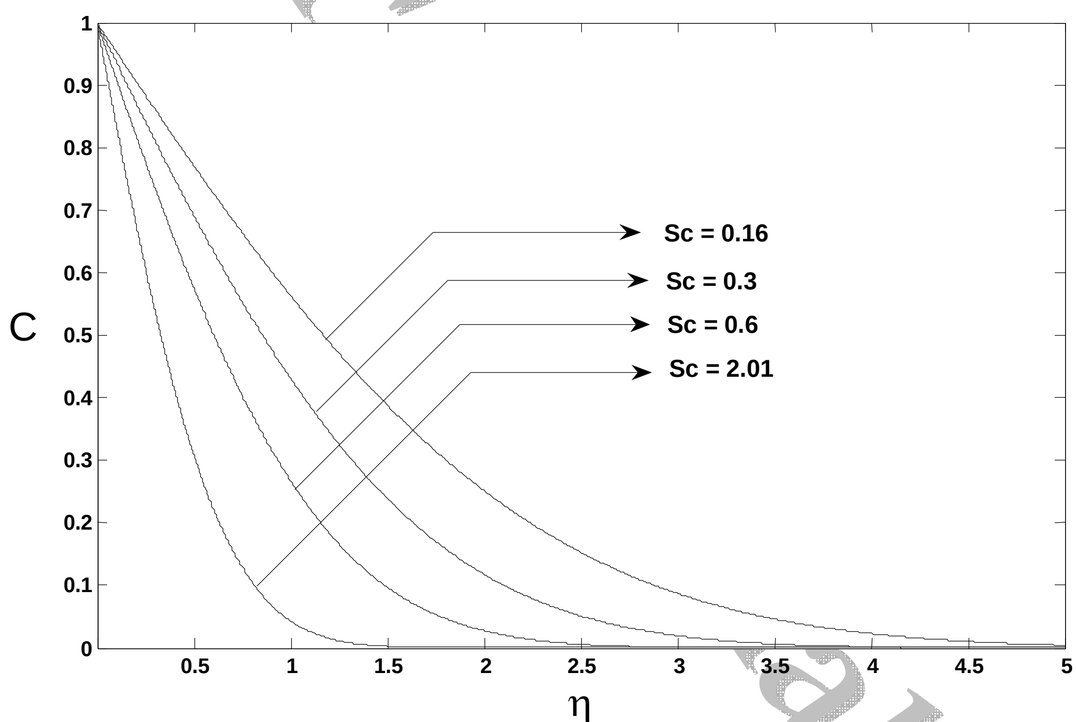

Figure 1. Concentration profiles for different values of Sc

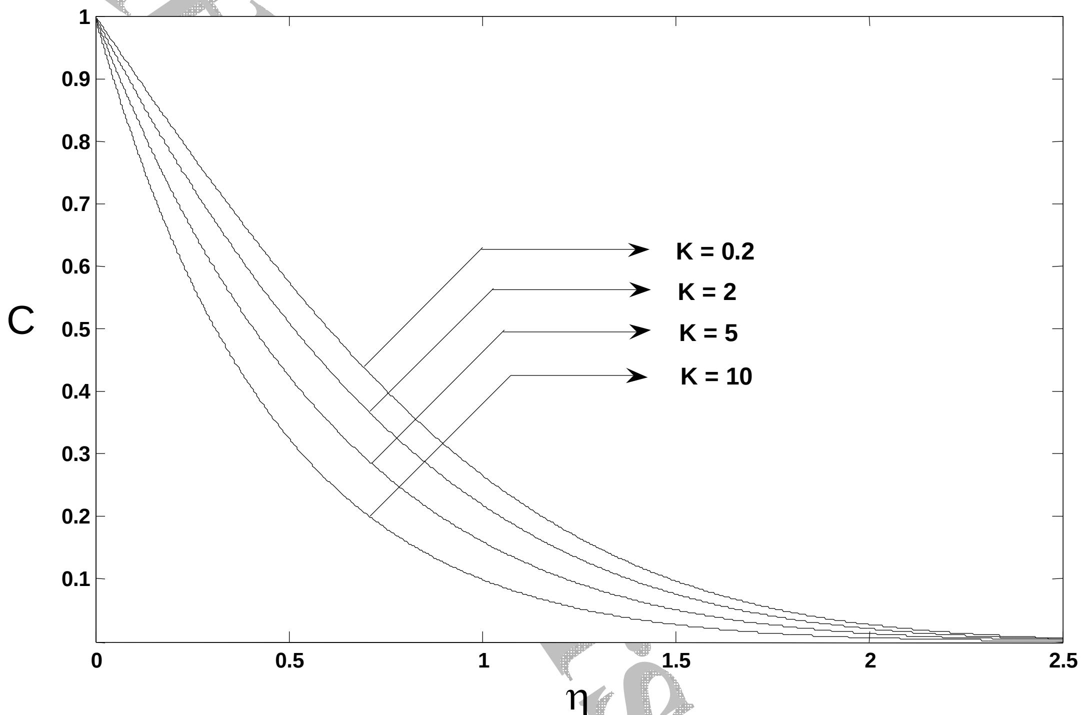

Figure 2. Concentration profiles for different values of K

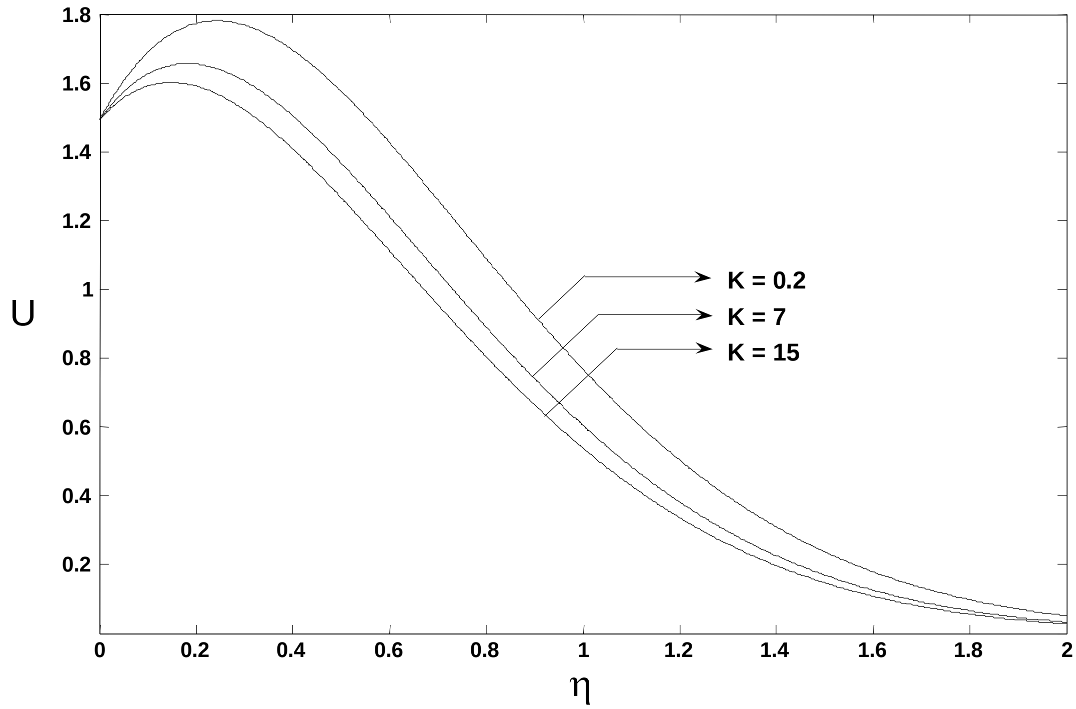

Figure 3.Velocity profiles for different values of K

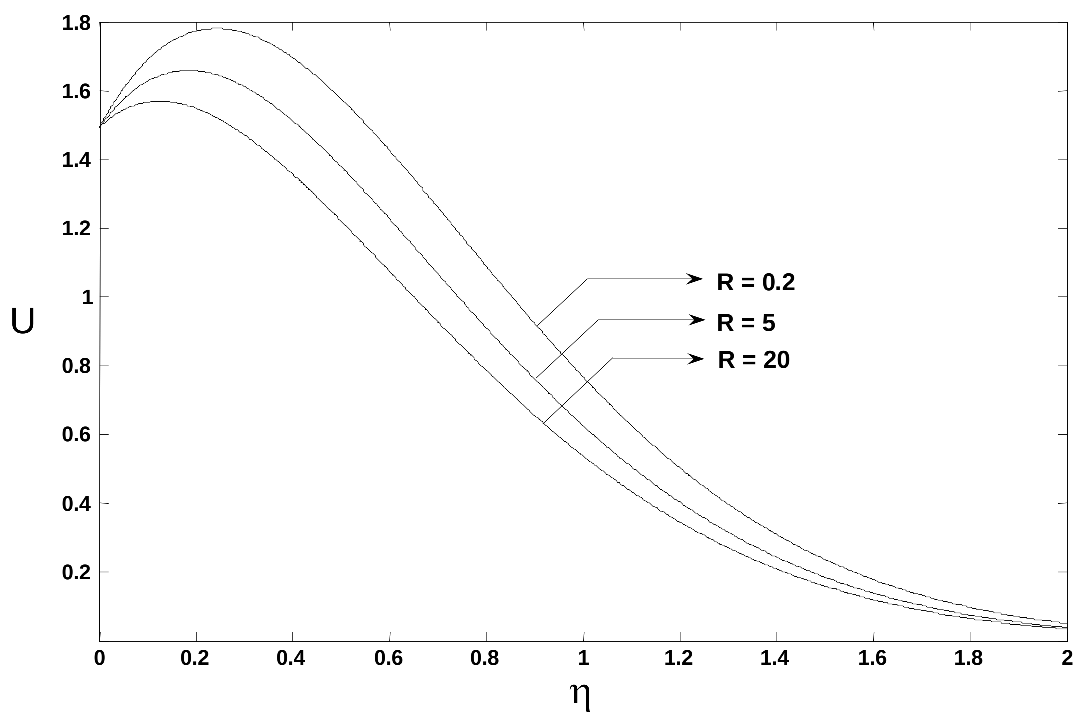

Figure 4. Velocity profiles for different values of R

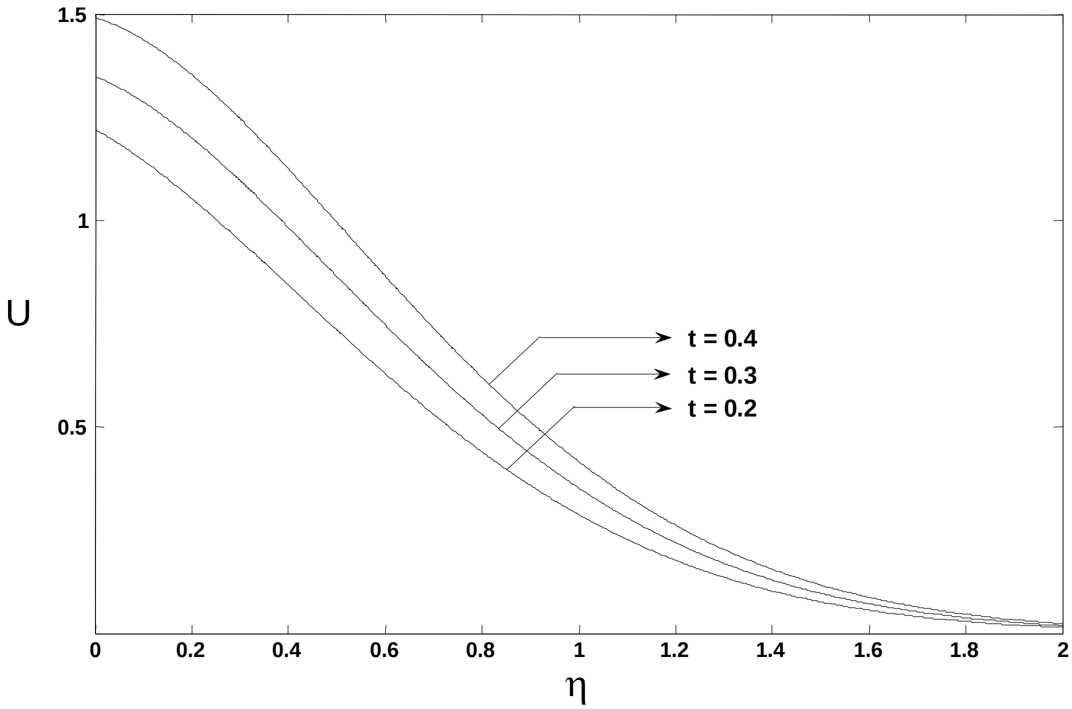

Figure 5. Velocity profiles for different values of t

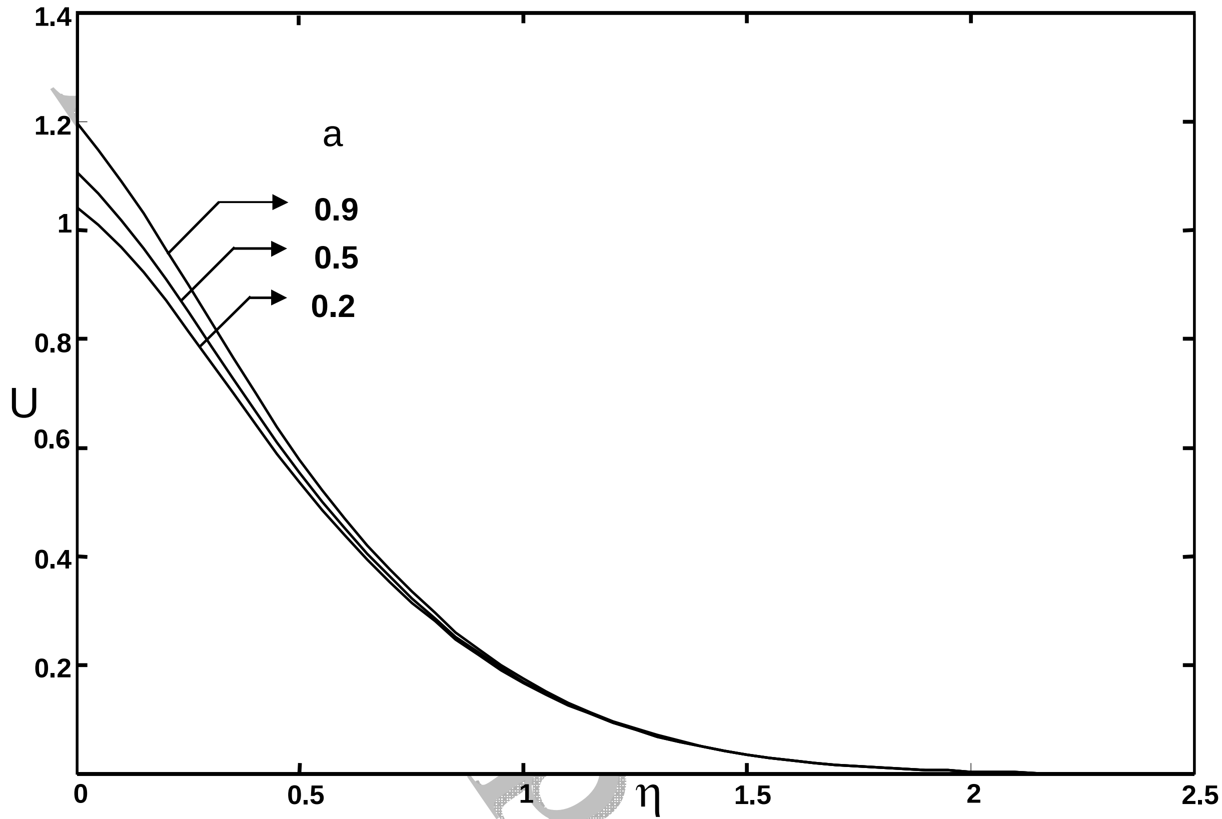

Figure 6. Velocity profiles for different a

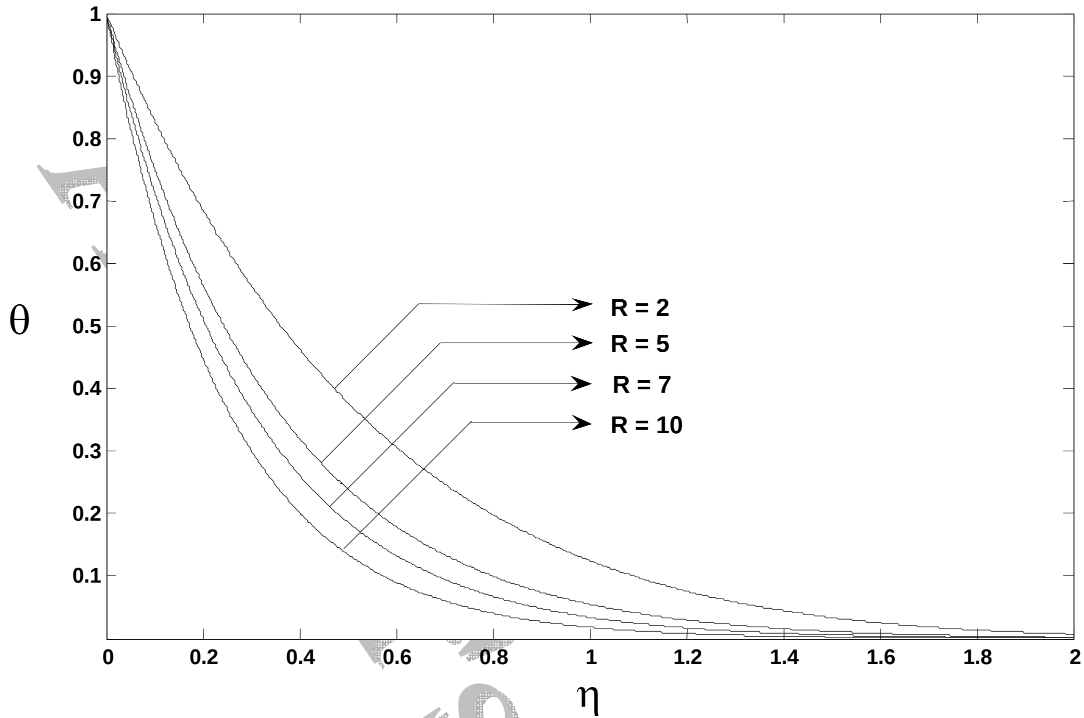

Figure 7. Temperature profiles for different values of R