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FLOW PAST AN ACCELERATED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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Abstract

An analysis of the unsteady flow of an incompressible fluid past an uniformly accelerated infinite vertical plate has been presented in the presence of variable temperature and mass diffusion. The temperature of the plate as well as concentration near the plate are made to raise linearly with time. The dimensionless governing equations are tackled using Laplace-transform technique. The velocity, the temperature and the concentration fields are studied for different physical parameters like Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing values of the Schmidt number.

Keywords: linear, accelerated, vertical plate, heat transfer, mass diffusion,unsteady.
Mathematics Subject Classification : 76R10

1. Introduction

The problem of free convective flow past an uniformly accelerated vertical plate has many technological applications such as filtration processes, the drying of porous materials in textile industries and saturation of porous materials by chemicals. The combined effect of heat and mass transfer plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, spacecraft design, solar energy collectors, design of chemical processing equipment, satellites and space vehicles are examples of such engineering applications.

Gupta et al [1] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[2] extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis al[3]. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar[4]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh[5]. Basant Kumar Jha and Ravindra Prasad[6] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo[7].

Hence, it is proposed to study the effects of on flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Mathematical Analysis

The unsteady flow of a viscous incompressible fluid past an uniformly accelerated vertical infinite plate with variable temperature and mass diffusion has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3 t'}{\nu}$ in its own plane and the temperature from the plate is raised linearly with respect time and and the concentration level near the plate is also raised linearly with time t. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \textit{for all} & y, t' \leq 0 \\ t' > 0: & u = \frac{u_0^3 t'}{\nu}, & T = T_\infty + (T_w - T_\infty)\, A t', & C' = C'_\infty + (C'_w - C'_\infty)\, A t' & \textit{at} & y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \textit{as} & y \to \infty \end{array} \tag{4}$$

Where, $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, t = \frac{u_0^2 t'}{\nu}, Y = \frac{y u_0}{\nu},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \tag{5}$$

$$Gc = \frac{g\nu\beta^*(C'_w - C'_\infty)}{u_0^3},\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D}$$

in equations (1) to (4), lead to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0 & for\ all \quad Y, t \le 0 \\ t > 0: & U = t, & \theta = t, & C = t & at \quad Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & as \quad Y \to \infty \end{array} \tag{9}$$

3. Solution Procedure

The dimensionless governing equations (6) to (8), subject to the initial and boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\,\eta^2\ Pr)\,erfc(\eta\sqrt{Pr}) - \frac{2}{\sqrt{\pi}}\eta\ \sqrt{Pr}\ \exp(-\eta^2 Pr)\right] \tag{10}$$

$$C = t\left[(1 + 2\,\eta^2\ Sc)\,erfc(\eta\sqrt{Sc}) - \frac{2\eta}{\sqrt{\pi}}\sqrt{Sc}\ \exp(-\eta^2 Sc)\right] \tag{11}$$

$$\begin{aligned} U = {} & t\left[(1 + 2\,\eta^2)\,erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\ & + \frac{Gr\,t^2}{6(Pr-1)}\Bigg[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2) \\ & - (3 + 12\eta^2\ \mathrm{Pr} + 4\eta^4(\mathrm{Pr})^2)\,erfc(\eta\sqrt{\mathrm{Pr}}) + \frac{\eta\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}(10 + 4\eta^2\ \mathrm{Pr})\exp(-\eta^2\ \mathrm{Pr})\Bigg] \\ & + \frac{Gc\,t^2}{6(Sc-1)}\Bigg[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2) \\ & - (3 + 12\eta^2 Sc + 4\eta^4(Sc)^2)\,erfc(\eta\sqrt{Sc}) + \frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10 + 4\eta^2 Sc)\exp(-\eta^2 Sc)\Bigg] \end{aligned}$$

(12)

Where, $\eta = \frac{Y}{2\sqrt{t}}$.

4. Results and Discussion

For the purpose of physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. The the value of the Prandtl number Pr is chosen such that it represent air($Pr = 0.71$). The numerical values of the velocity, the temperature and the concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of velocity for different values of the Schmidt number ($Sc = 0.16, 0.6, 2.01$), $Gr = Gc = 5$ and time $t = 0.2$ are shown in figure 1. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number. The velocity profiles for different ($t = 0.2, 0.4, 0.6, 0.8$), $Gr = 5$ and $Gc = 5$ are studied and presented in figure 2. It is observed that the velocity increases with increasing values of t. Figure 3. demonstrates the effects of different thermal Grashof number ($Gr = 2, 5$) and mass Grashof number ($Gc = 2, 5$) on the velocity at time $t = 0.6$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The temperature profiles for different time are calculated for air from Equation (10) and these are shown in Figure 4. It is observed that the temperature increases with increasing values of time t.

Figure 5 represents the effect of concentration profiles for different Schmidt number($Sc = 0.16, 0.6, 2.01$)and time $t =$. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

5. Concluding Remarks

An exact solution of unsteady flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and mass diffusion have been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, Schmidt number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the Schmidt number. It is also observed that the wall concentration increases with decreasing values of the Schmidt number.

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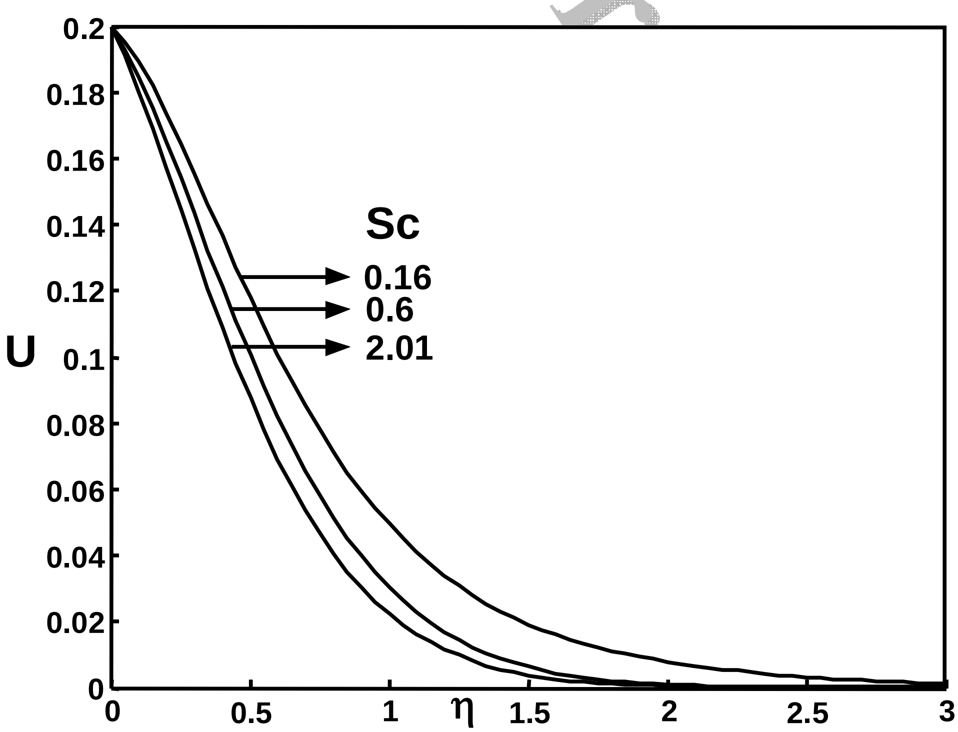

Figure 1. Velocity profiles for different values of Sc

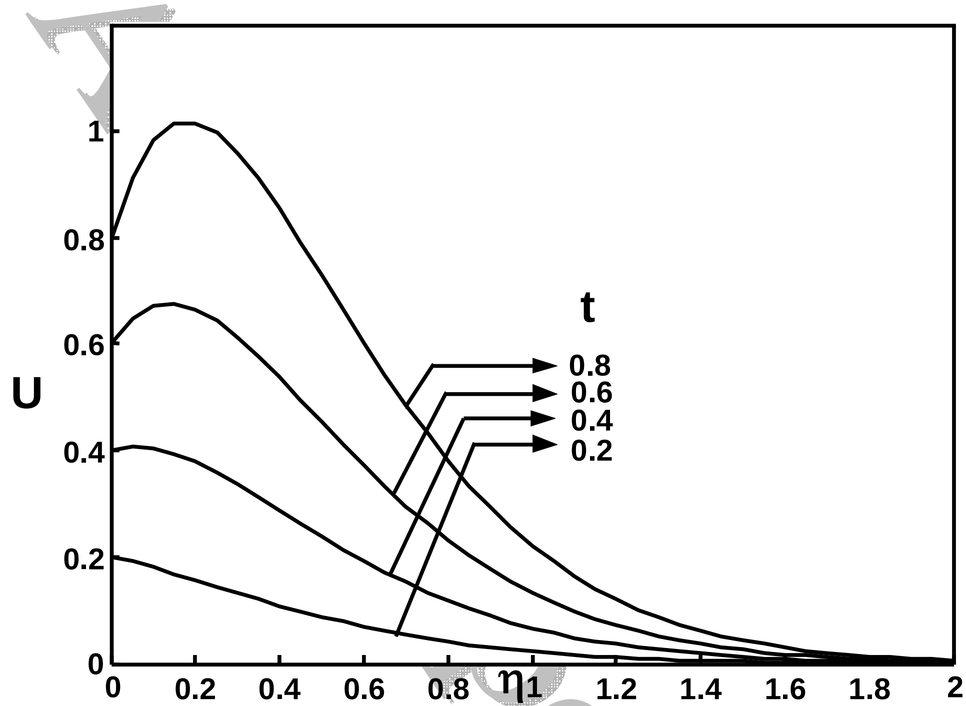

Figure 2. Velocity profiles for different values of t

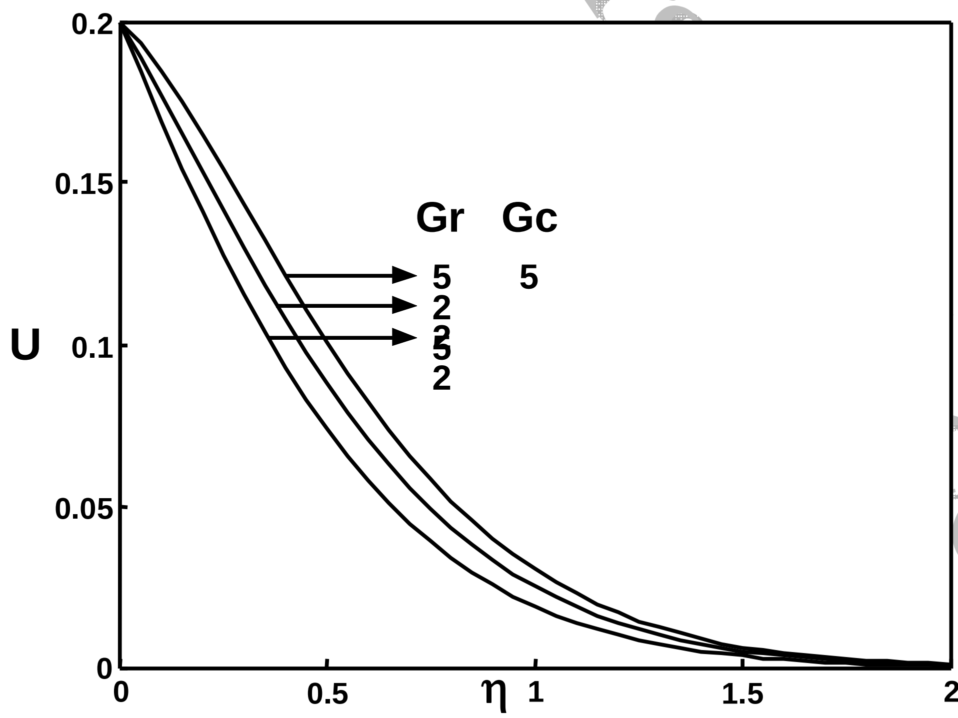

Figure 3. Velocity profiles for different values of Gr and Gc

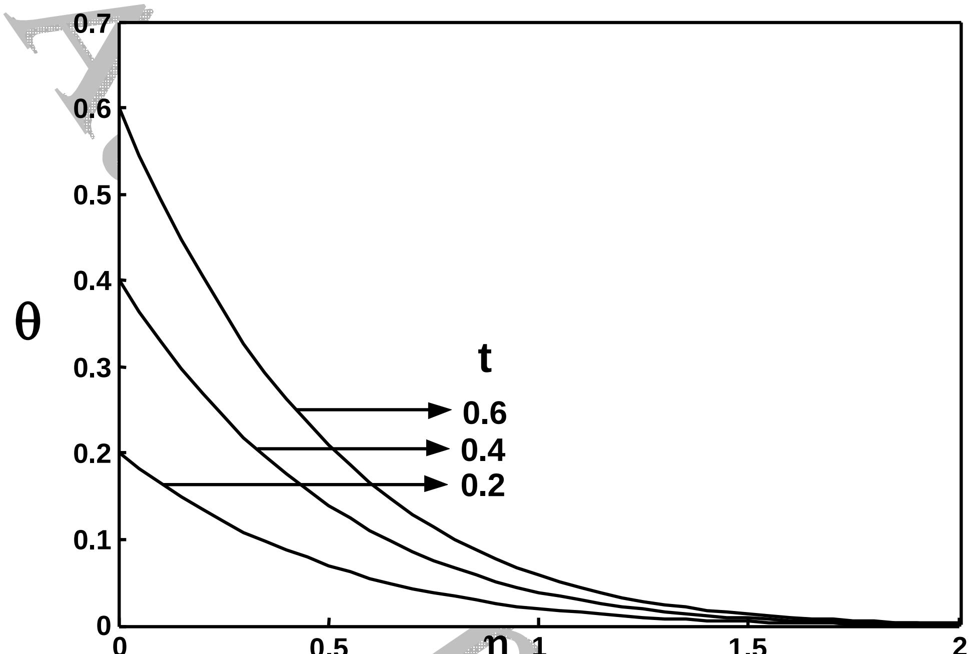

Figure 4. Temperature profiles for different values of t

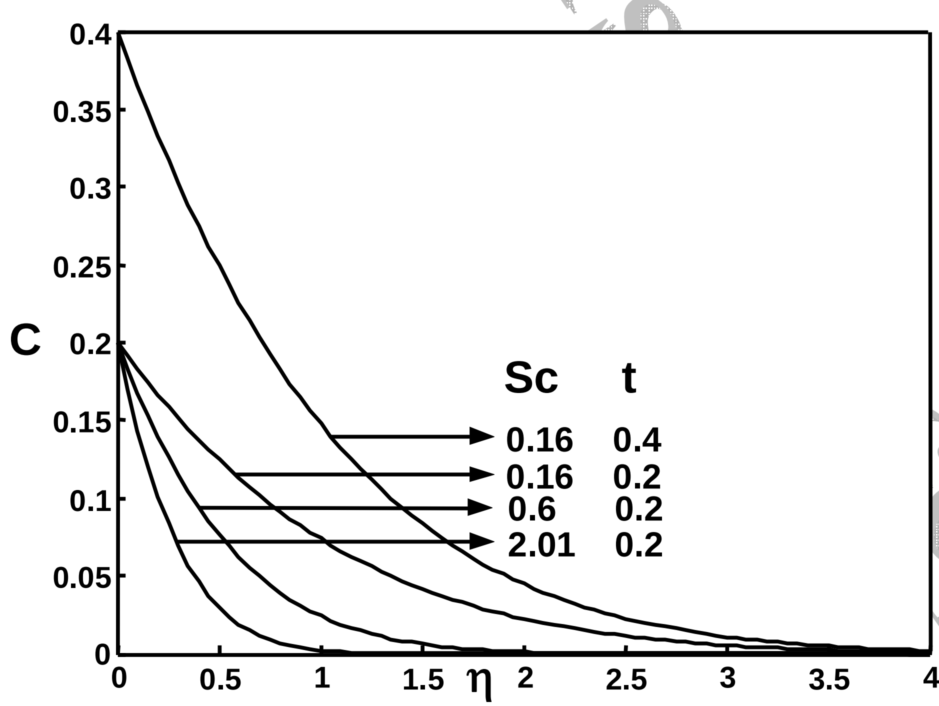

Figure 5. Conentration profiles for different values of Sc & t