

**REFLECTION OF LONGITUDINAL MICRO-ROTATIONAL WAVE  
FROM A FIXED MICRO-ELASTIC SURFACE**

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**ABSTRACT:**

Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface is studied and obtained the amplitude ratios of micro-rotation, transverse displacement and transverse micro-rotation waves. The result of micropolar theory is obtained as a particular case of it.

**KEY WORDS:** Reflection – micro-rotational wave -- micro-elastic surface.

**INTRODUCTION:**

The theory of micromorphic materials was developed by Eringen[1]. This theory is simplified by Koh[2] by extending the concept of coincidence of the principal directions of the stress and strain of classical elasticity to theory of micromorphic materials and called it as theory of micro-isotropic, micro-elastic solids. It consists of 12 deformation fields namely, 3 macro-displacements, 3 micro-rotations and 6 micro-strains. Further it has 10 elastic constants. If an element  $\Delta V + \Delta S$  contains N discrete micro-material elements  $\Delta V^{(\alpha)} + \Delta S^{(\alpha)}$ , ( $\alpha=1,2,\dots, N$ ), then the micro-rotational and micro – strains are the rotation and strain of micro elements  $\Delta V^{(\alpha)} + \Delta S^{(\alpha)}$  about the centre of mass of  $\Delta V + \Delta S$ .

Eringen[3] have investigated the problem of plane waves from the flat boundary of a micropolar elastic half space and obtained the amplitude ratios of different reflected waves. Reflection and refraction of a longitudinal micro-rotational wave at an interface between two micropolar elastic media in welded contact is investigated by Tomar and Gonga[4]. In the present work the amplitude ratios of micro-rotation waves and transverse displacement waves at a fixed micro-isotropic, micro-elastic surface are obtained.

## BASIC EQUATIONS:

The micro-displacement in the micro-elastic continuum is denoted by  $u_k$  and micro-deformation by  $\phi_{mn}$ . Further the macro-strain  $e_{km} = u_{(k,m)}$ , the macro-rotation vector  $r_k = \frac{1}{2} \epsilon_{kmn} u_{n,m}$ , the micro-strain  $\phi_{(mn)}$  and the micro-rotation vector  $\phi_p = \frac{1}{2} \epsilon_{pkm} \phi_{k,m}$ , where ( ) denotes the symmetric part, comma indicates differentiation with respect to the coordinate  $x_k$ .

The stress measures are the asymmetric stress (macro stress)  $t_{km}$ , the relative stress (micro-stress)  $\sigma_{km}$  and stress moment  $t_{kmn}$ . The couple stress tensor  $m_{kl}$  is defined by  $m_{kl} = \epsilon_{pnm} t_{kmn}$ .

The constitutive equations for micro-isotropic, micro-elastic medium are given by [2,5].

$$t_{(km)} = A_1 e_{pp} \delta_{km} + 2A_2 e_{km} \quad (1)$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \epsilon_{pkm} (r_p + \phi_p) \quad (2)$$

$$\sigma_{[km]} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)} \quad (3)$$

$$t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(mn),k} \quad (4)$$

$$m_{kl} = -2(B_3 \phi_{l,k} + B_4 \phi_{k,l} + B_5 \phi_{p,p} \delta_{kl}) \quad (5)$$

where [ ] denotes the anti-symmetric part and

$$\begin{aligned} A_1 &= \lambda + \sigma_1, & B_1 &= \tau_3 \\ A_2 &= \mu + \sigma_2, & 2B_2 &= \tau_7 + \tau_{10} \\ A_3 &= \sigma_5, & B_3 &= 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10} \\ A_4 &= -\sigma_1, & B_4 &= -2\tau_4 \\ A_5 &= -\sigma_2, & B_5 &= -2\tau_9 \end{aligned} \quad (6)$$

where  $\lambda, \mu, \sigma_1, \sigma_2, \sigma_5, \tau_3, \tau_4, \tau_7, \tau_9$  and  $\tau_{10}$  are elastic moduli.

The displacement equations of motion for micro-isotropic, micro-elastic body occupying a region R are given by

$$(A_1 + A_2 - A_3)u_{p,pp} + (A_2 + A_3)u_{m,pp} + 2A_3 \epsilon_{pkm} \phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2} \quad (7)$$

$$B_1 \phi_{pp,kk} \delta_{ij} + 2B_2 \phi_{(ij),kk} - A_4 \phi_{pp} \delta_{ij} - 2A_5 \phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2} \rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2} \quad (8)$$

$$2B_3 \phi_{p,mm} + 2(B_4 + B_5) \phi_{m,mp} - 4A_3 (r_p + \phi_p) - \rho l_p = \rho j \frac{\partial^2 \phi_p}{\partial t^2} \quad (9)$$

where  $\rho$  is the average mass density,  $f_m$  is the body force per unit mass,  $f_{(ij)}$  is the symmetric body moment per unit mass,  $j$  is the micro-inertia,  $l_p$  is the body couple per unit mass.

### Reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic solid

K. Somaiah and K. Sambaiah[6] have shown that there are 12 waves propagate in an infinite micro-isotropic, micro-elastic solid with six different velocities.

a) A longitudinal displacement wave propagating with speed  $v_1$  where

$$v_1^2 = \frac{A_1 + 2A_2}{\rho} \quad (10)$$

b) A longitudinal micro-rotation wave with speed  $v_2$  where

$$v_2^2 = \frac{2(B_3 + B_4 + B_5)}{\rho j \left(1 - \frac{\omega_1^2}{\omega^2}\right)} \quad (11)$$

with a cut-off frequency

$$\omega_1^2 = \frac{4A_3}{\rho j} \quad (12)$$

c) Two sets of coupled transverse displacement and transverse micro-rotation waves with speeds  $v_3$  and  $v_4$  where

$$v_3^2 = \frac{1}{2 \left(1 - \frac{2\omega_0^2}{\omega^2}\right)} \left[ c_4^2 + c_2^2 \left(1 - \frac{2\omega_0^2}{\omega^2}\right) + c_3^2 \left(1 - \frac{3\omega_0^2}{\omega^2}\right) + \left[ c_4^2 - c_2^2 - c_3^2 + 2 \left( c_2^2 + \frac{3}{2} c_3^2 \right) \frac{\omega_0^2}{\omega^2} - 4c_3^2 c_4^2 \frac{\omega_0^2}{\omega^2} \right]^{1/2} \right] \quad (13)$$

$$v_4^2 = \frac{1}{2 \left(1 - \frac{2\omega_0^2}{\omega^2}\right)} \left[ c_4^2 + c_2^2 \left(1 - \frac{2\omega_0^2}{\omega^2}\right) + c_3^2 \left(1 - \frac{3\omega_0^2}{\omega^2}\right) \right]$$

$$-\left[ c_4^2 - c_2^2 - c_3^2 + 2\left( c_2^2 + \frac{3}{2}c_3^2 \right) \frac{\omega_0^2}{\omega^2} - 4c_3^2 c_4^2 \frac{\omega_0^2}{\omega^2} \right]^{1/2} \quad (14)$$

with  $\sqrt{2} \omega_0$  as cut-off frequency .

d) Six waves corresponding to micro-strains and they propagate with two distinct velocities.

We investigate the reflection of longitudinal micro-rotation wave from a fixed micro-isotropic, micro-elastic surface. Eringen[3] shown that the incident and reflected waves propagate in the same plane. We select this plane to be the  $z=0$  plane.

Thus we take

$$\vec{u} = (u, 0, 0) \quad (15)$$

$$\vec{\phi} = (0, \phi_2, \phi_3) \quad (16)$$

where  $u, \phi_2, \phi_3$  are functions of  $y, z$  and  $t$ . The boundary conditions are  $u=0, \phi_2=0, \phi_3=0$  at  $z=0$ .

We decompose the vector  $\vec{u}$  and  $\vec{\phi}$  in to scalar and vector potentials as

$$\vec{u} = \nabla G + \nabla \times \vec{H}, \quad \nabla \cdot \vec{H} = 0 \quad (17)$$

$$\vec{\phi} = \nabla R + \nabla \times \vec{S}, \quad \nabla \cdot \vec{S} = 0 \quad (18)$$

Substituting (17), (18) into (7) and (8), we get that these equations are satisfied if

$$(c_1^2 + c_3^2) \nabla^2 G = 0 \quad (19)$$

$$(c_4^2 + c_5^2) \nabla^2 R - 2\omega_0^2 R = 0 \quad (20)$$

$$(c_2^2 + c_3^2) \nabla^2 \vec{H} + c_3^2 \nabla \times \vec{S} = 0 \quad (21)$$

$$c_4^2 \nabla^2 \vec{S} - 2\omega_0^2 \vec{S} - \omega_0^2 \times \vec{H} = 0 \quad (22)$$

where

$$c_1^2 = \frac{A_1 + 2A_2 - 2A_3}{\rho}, \quad c_2^2 = \frac{A_2 - A_3}{\rho} \quad (23)$$

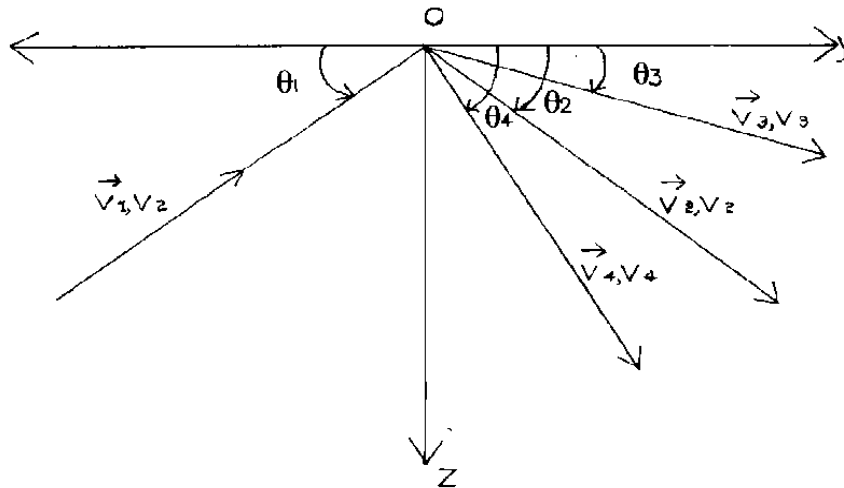
$$c_3^2 = \frac{2A_3}{\rho}, \quad c_4^2 = \frac{2B_3}{\rho j}, \quad c_5^2 = \frac{2(B_4 + B_5)}{\rho j}, \quad \omega_0^2 = \frac{2A_3}{\rho j} = \frac{c_3^2}{j}$$

Consider an incident longitudinal micro-rotation wave travelling with speed  $v_2$  in the direction  $\vec{v}_1$  making an angle  $\theta_1$  with the plane boundary give rise to

- A reflected longitudinal micro-rotation wave travelling with speed  $v_2$  in the direction  $\vec{v}_2$  and making an angle  $\theta_2$  with the fixed surface.
- A set of reflected waves travelling with speed  $v_3$  in the direction  $\vec{v}_3$  and making an angle  $\theta_3$  with the surface.

c) A similar set of reflected waves travelling with speed  $v_4$  in the direction  $\hat{p}_4$  and making an angle  $\theta_4$  with the fixed surface.

The geometry of the problem is shown in fig.1



In thi

$$R = a_1 \exp[ik \dots]$$

$$H_p = \left[ A_{py} J - \left( \frac{\dots}{\sin \theta_p} \right) A_{py} k \right] \exp[ik \{ \cos \theta_p y + \sin \theta_p z \} - i \omega_p t] \tag{24}$$

$$S_p = i \frac{\rho \beta_{Ap}}{\beta_{Bp}} \left( \frac{A_{py}}{\sin \theta_p} \right) \exp[ik_p \{ \cos \theta_p y + \sin \theta_p z \} - i \omega_p t]$$

where  $p = 3,4$ ;  $a_1, a_2$  are the amplitudes of incident and reflected longitudinal micro-rotation waves respectively;  $A_{py}$  are the amplitudes of reflected coupled waves.

$$\beta_{Ap} = \left[ \frac{i \omega_0^2}{k_p \left( v_p^2 - 2 \frac{\omega_0^2}{k_p^2} - c_4^2 \right)} \right] \beta_{Bp} \tag{25}$$

$$\omega_i = k_i v_i, \quad (i = 2,3,4)$$

$\omega_i$  is angular frequency and  $k_i$  is the wave number.

In view of (17) and (18), the boundary conditions become

$$u = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = 0$$

$$\phi_2 = \frac{\partial R}{\partial y} + \frac{\partial S}{\partial z} = 0 \tag{26}$$

$$\phi_3 = \frac{\partial R}{\partial y} - \frac{\partial S}{\partial z} = 0$$

Eringen[7] has shown that

$$\frac{\cos \theta_1}{v_2} = \frac{\cos \theta_3}{v_3} = \frac{\cos \theta_4}{v_4} \text{ and } \theta_2 = \theta_1 \quad (27)$$

$$\text{and } \omega_2 = \omega_3 = \omega_4 = \omega \text{ (say)} \quad (28)$$

Substituting (24) into (26) and using (27) and (28) we obtain

$$\sin \theta_4 z_2 + \sin \theta_3 z_3 = 0 \quad (29)$$

$$\cos \theta_2 z_1 + \frac{\beta_{A3}}{k_2 \beta_{B3}} z_2 + \frac{\beta_{A4}}{k_2 \beta_{B4}} z_3 = -\cos \theta_1 \quad (30)$$

$$\sin \theta_2 z_1 - \frac{\cos \theta_3}{\sin \theta_3} \frac{\beta_{A3}}{k_2 \beta_{B3}} z_2 - \frac{\cos \theta_4}{\sin \theta_4} \frac{\beta_{A4}}{B_{B4} k_2} z_3 = \sin \theta_1 \quad (31)$$

$$\text{where } z_1 = \frac{a_2}{a_1}, \quad z_2 = \frac{k_3 A_{3y}}{a_1}, \quad z_4 = \frac{k_4 A_{4y}}{a_1} \quad (32)$$

Solving the equations (30) to (32) we obtain the amplitude ratios and they are given by

$$z_1 = \frac{1}{\Delta} \left[ \frac{\beta_{A3}}{\beta_{B3}} \frac{1}{k_2} \cos(\theta_1 + \theta_3) - \frac{\beta_{A4}}{\beta_{B4}} \frac{1}{k_2} \cos(\theta_1 + \theta_4) \right] \quad (33)$$

$$z_2 = \frac{1}{\Delta} \sin \theta_3 \sin(\theta_1 + \theta_2) \quad (34)$$

$$z_3 = \frac{1}{\Delta} \sin \theta_4 \sin(\theta_1 + \theta_2) \quad (35)$$

$$\text{where } \Delta = \frac{\beta_{A4}}{\beta_{B4}} \frac{1}{k_2} \cos(\theta_2 - \theta_4) - \frac{\beta_{A3}}{\beta_{B3}} \frac{1}{k_2} \cos(\theta_2 - \theta_3)$$

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