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Effects Of Non-Uniform Slot Injection (Suction) Into Two Dimensional Mhd Boundary Layer Flow With Temperature Dependent Viscosity

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ABSTRACT

In this paper, we study the effect of slot injection (suction) into a steady, two-dimensional laminar boundary layer flow with temperature dependent viscosity and an applied magnetic field. The fluid viscosity is assumed to vary as inverse linear function of temperature. Non-similar solutions have been obtained from the starting point of the streamwise co-ordiante to the exact point of separation. The difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome by applying an implicit finite difference scheme with the quasi-linearization technique and an appropriate selection of finer step size along the streamwise direction. Numerical results are presented graphically to display the effect of temperature dependent viscosity, magnetic field and slot injection (suction) on skin friction and heat transfer coefficients. The results indicate that magnetic field and temperature dependent viscosity increases both skin friction and heat transfer coefficients. And, it is found that the boundary layer separation can be delayed by non-uniform slot suction and also by moving the slot downstream whereas, non-uniform slot injection does the reverse effect. Further, in the case of suction the momentum as well as thermal boundary layer thickness decrease while the slot injection does the opposite.

Keywords: Magnetic field, temperature dependent viscosity, skin friction, heat transfer, Slot injection (suction)

INTRODUCTION

In many technological solutions, mass transfer through a wall slot (i.e mass transfer occurs in a small porous section of the body surface while there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for the various potential applications including thermal protection, energizing for the inner portion of the boundary layer in adverse pressure gradient, and skin friction reduction on control surfaces. Moreover mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or atleast delay separation of the viscous region. Several investigators [1-4] have studied the effect of slot injection (suction) into a laminar compressible boundary layer by considering the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuities at the leading and the trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer distribution along a streamwise slot as has been discussed in Minkowycz.et.al. [5].

It is well known that many boundary layer flow and heat transfer problems of current practical interest do not admit similarity transformations. A detailed analysis of the flow situation in the laminar boundary layer taking non-similarity into account is of prime importance, where the non-similarity may be due to the free stream velocity or due to the curvature of the body or due to the surface mass transfer or due to all these effects. A review of non-similarity solution methods along with citations of some relevant publications is given by Dewey and Gross [6]. Since then several researches [7-9] have attempted to study the behavior of non-similar boundary layer flows.

Several studies [10-12] have been made to investigate the effect of temperature-dependent viscosity on laminar boundary layer flows for various situations. The viscosity of a fluid is an important property in the analysis of fluid motion near solid boundaries and is of practical importance in many diverse applications. Since this physical property may change significantly with temperature, it is essential to consider flow and heat transfer problems assumig temperature dependent viscosity.

The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers [13-15] due to its applications in many scientific and engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions.

The purpose of the present investigation is to study the effect of non-uniform slot injection (suction) (i.e., mass transfer in a small porous section of the body surface and the remaining part of the body surface is solid) on the steady boundary layer flow over a two-dimensional body (cylinder) with an applied magnetic field and temperature dependent viscosity. The non-similar solutions have been obtained starting from the origin of the streamwise coordinate to the point of separation (zero skin friction in the streamwise direction) using an implicit finite difference scheme along with quasilinearization technique.

ANALYSIS

Consider the steady, laminar non similar boundary layer forced convection flow of an electrically conducting fluid over a two-dimensional body (cylinder) when the free stream velocity and non-uniform mass transfer (slot injection/suction) vary with the axial distance along the surface. Let x and y be the curvilinear coordinates along and perpendicular to the boundary, respectively, u and v be the corresponding velocity components. The contour of the body of revolution is specified by the radii $r(x)$ of the section perpendicular to the axis (Fig.1). A magnetic field B_0 is applied in y direction normal to the body surface and it is assumed that magnetic Reynolds number is small. The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature.

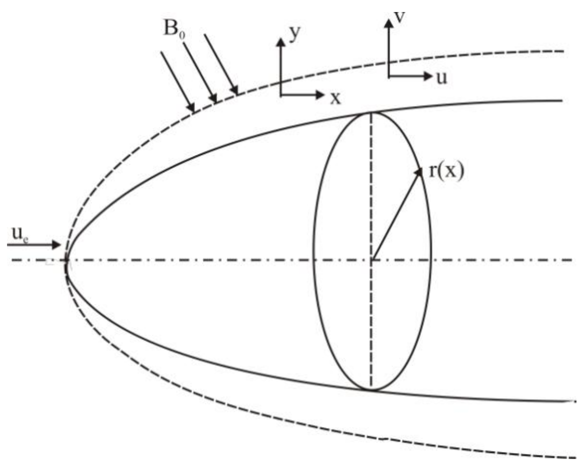

Fig.1 Flow model and co-ordinate system

The fluid is assumed to have constant physical properties except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) (see Lai and Kulacki [16]), viz.,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma(T-T_\infty)\right]; \quad \frac{1}{\mu}=a(T-T_e) \quad \text{where } a=\frac{\gamma}{\mu_\infty}; \quad T_e=T_\infty-\frac{1}{\gamma} \tag{1}$$

Here, both $'a'$ and T_e are constants and their values depend on the reference state and the thermal property of the fluid i.e., (γ) . In general, $a>0$ for liquids and $a<0$ for gases.

Neglecting the effects of transverse curvature, the boundary layer equations governing the flow are given by :

$$\frac{\partial}{\partial x}(r^j u)+\frac{\partial}{\partial y}(r^j v)=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0{}^2}{\rho}(u-u_e) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

with boundary conditions:

$$u(x,0)=0,\quad v(x,0)=v_w(x),\quad T(x,0)=T_w=\text{ constant} \tag{5}$$
$$u(x,\infty)=u_e(x),\quad T(x,\infty)=T_\infty=\text{ constant}$$

Applying the following transformations:

$$\xi=\int_0^x\frac{u_e}{u_\infty}\left(\frac{r}{L}\right)^{2j}d\left(\frac{x}{L}\right);\qquad \eta=\frac{u_e}{u_\infty}\left(\frac{\text{Re}_L}{2\xi}\right)^{1/2}\frac{y}{L}\left(\frac{r}{L}\right)^j;\qquad \psi(x,y)=u_\infty L\left(\frac{2\xi}{\text{Re}_L}\right)^{1/2}f(\xi,\eta);$$
$$u=\left(\frac{L}{r}\right)^j\psi_y;\quad v=-\left(\frac{L}{r}\right)^j\psi_x;\quad T=T_\infty+(T_w-T_\infty)G(\xi,\eta) \tag{6}$$

to Eqns. (2) – (4), we see that the continuity Eqn. (2) is identically satisfied and Eqns. (3) and (4) reduces to non-dimensional form, respectively, as:

$$F''+\beta(\xi)\left(1-\frac{G}{Ge}\right)(1-F^2)+\left(1-\frac{G}{Ge}\right)f\,F'+\left(\frac{G'}{Ge-G}\right)F'-PM\,(F-1)\left(1-\frac{G}{Ge}\right)=\left(1-\frac{G}{Ge}\right)2\xi\left(F\,F_\xi-F'\,f_\xi\right) \tag{7}$$

$$\text{Pr}^{-1}G''+f\,G'=2\xi\left(FG_\xi-G'f_\xi\right) \tag{8}$$

where

$$\frac{u}{u_e}=f'=F;\quad v=-\left(\frac{r}{L}\right)^j u_e(2\xi\,\text{Re}_L)^{-1/2}\left(f+2\xi\,f_\xi+F\eta(\beta+\alpha_1-1)\right);$$

$$\beta(\xi)=\left(\frac{2\xi}{u_e}\right)\left(\frac{du_e}{d\xi}\right);\quad \alpha_1=\left(\frac{2\xi\,j}{r}\right)\left(\frac{dr}{d\xi}\right);\quad M=\left(\frac{2}{3}\right)\frac{\sigma B_0^2 L}{\rho u_\infty};\qquad P=3\xi\left(\frac{L}{r}\right)^{2j}\left(\frac{u_\infty}{u_e}\right)^2;$$

$$\text{Re}_L=\frac{\rho u_\infty L}{\mu};\quad \text{Pr}=\frac{\nu}{\alpha};\ f=\int_0^\eta F\,d\eta+f_w\ \text{ and }\ f_w=-\xi^{-1/2}\left(\frac{\text{Re}_L}{2}\right)^{1/2}\int_0^x\left(\frac{v_w}{u_\infty}\right)\left(\frac{r}{L}\right)^j d\left(\frac{x}{L}\right) \tag{9}$$

Here ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimensionless stream functions, respectively; Pr is the Prandtl number; ρ,ν,α are respectively density, kinetic viscosity and thermal diffusivity; F is the dimensionless velocity; T and G are dimensional and dimensionless temperature, respectively; L is the characteristic length; R is the radius of the cylinder, Re_L is the Reynolds number ; f_w is the surface mass transfer; α_1 is a dimensionless parameter; β is the pressure gradient parameter; M is the nondimensional magnetic parameter. The subscripts ∞, e and w denote the conditions at the free stream, edge of the boundary layer and at the wall, respectively, and j = 0 for two-dimensional flow. $u_e(x)$ is the potential flow velocity and $v_w(x)$ denotes the surface mass transfer distribution. T_∞ is the constant temperature of the fluid maintained at the edge of the boundary layer and T_w is the uniform temperature of the body. The subscript ξ denote partial derivatives with respect to ξ and prime $(')$ denote derivates with respect to η.

The transformed boundary conditions are

$$F(\xi,0)=0,\;\; G(\xi,0)=1 \;\;\text{and}\;\; F(\xi,\infty)=1,\;\; G(\xi,\infty)=0 \tag{10}$$

The skin friction coefficient and, the heat transfer coefficient in the form of Nusselt number, can be expressed respectively as

$$C_f=\left(\frac{2Ge}{Ge-1}\right)\left(\frac{u_e}{u_\infty}\right)^2\left(\frac{r}{L}\right)^j\left(2\xi\,\mathrm{Re}_L\right)^{-1/2}F'_w \quad \text{and} \quad Nu=\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\left(\frac{r}{L}\right)^j\left(\frac{u_e}{u_\infty}\right)(-G'_w) \tag{11}$$

The dimensionless temperature G and viscosity ratio $\frac{\mu}{\mu_\infty}$ are re-defined as follows:

$$G=\frac{T-T_e}{T_w-T_\infty}+G_e \quad \text{and} \quad \text{hence} \quad \frac{\mu}{\mu_\infty}=\frac{G_e}{G_e-G} \tag{12}$$

where G_e is constant, called viscosity variation parameter, which is defined by

$$G_e=\frac{T_e-T_\infty}{T_w-T_\infty}=\frac{-1}{\gamma(T_w-T_\infty)} \;=\text{constant} \tag{13}$$

and its value is determined by viscosity characteristics of the fluid under consideration and operating temperature difference $\Delta T=T_w-T_\infty$.

It may be remarked here that, if G_e is large (i.e., $G_e\to\infty$), the effect of variable viscosity can be neglected. On the other hand, for a smaller value of G_e, either the fluid viscosity changes markedly with temperature or operating temperature difference is high. In either case, the variable viscosity effect is expected to become very significant. Also, it may be noted here that, liquid viscosity varies differently with temperature than that of gas and therefore, it is important to note that $G_e < 0$ for liquids and $G_e > 0$ for gases when the temperature difference ΔT is positive.

Equations (7) and (8) under conditions (10) can be solved numerically if β and u_e, which depend on the shape of the body, are prescribed. In particular, we have analyzed the effect of non-uniform slot injection (suction) into boundary layer flows over a cylinder. The free stream velocity distribution for the case of circular cylinder and the distance from the axis of the body are given by [17] :

$$\frac{u_e}{u_\infty}=2\sin(\bar{x}); \qquad \bar{x}=\frac{x}{R}; \quad j=0; \quad L=R \tag{14}$$

where $\bar{x}$ is the dimensionless distance along the surface which give rise non similarity in the flow. Consequently, the expressions for ξ, β, α_1, f_w are respectively given by

$$\xi = 2P_1; \qquad \beta = 2\cos(\bar{x})P_2^{-1}; \qquad P = \frac{3}{2(1+\cos\bar{x})}; \qquad \alpha_1 = 0; \tag{15}$$

$$f_w = \begin{cases} 0, & \bar{x} \le \bar{x}_0 \\ A(2P_1)^{-1/2} C(\bar{x}, \bar{x}_0), & \bar{x}_0 \le \bar{x} \le \bar{x}_0^{\,*} \\ A(2P_1)^{-1/2} C(\bar{x}_0^{\,*}, \bar{x}_0), & \bar{x} \ge \bar{x}_0^{\,*} \end{cases} \tag{16}$$

where the function $C(\bar{x}, \bar{x}_0) = 1 - \cos\{w^*(\bar{x} - \bar{x}_0)\}$; $P_1 = 1 - \cos\bar{x}$, $P_2 = 1 + \cos\bar{x}$ (17)

Here $v_w(x)$ in expression (9) is taken as

$$v_w = \begin{cases} -u_\infty \left(\frac{\mathrm{Re}_L}{2}\right)^{-1/2} A w^* \sin\{w^*(\bar{x} - \bar{x}_0)\}, & \bar{x}_0 \le \bar{x} \le \bar{x}_0^{\,*} \\ 0, & \bar{x} \le \bar{x}_0 \quad or\ \bar{x} \ge \bar{x}_0^{\,*} \end{cases} \tag{18}$$

where w^* and $\bar{x}_0$ are the two free parameters which determine the slot length and slot location. The function v_w is continuous for all values of $\bar{x}$ and it has a non-zero value only in the interval ($\bar{x}_0, \bar{x}_0^{\,*}$). The reason for taking such a type of function is that it allows the mass transfer to change slowly in the neighbourhood of the leading and trailing edges of the slot. The mass transfer parameter A > 0 for suction and A < 0 for injection.

It is convenient to express Eqns. (7) and (8) in terms of $\bar{x}$ instead of ξ. Equation (15) gives the relation between ξ and $\bar{x}$ as

$$\xi \frac{\partial}{\partial \xi} = S(\bar{x}) \frac{\partial}{\partial \bar{x}} \qquad \text{where } S(\bar{x}) = \tan\left(\bar{x}/2\right) \tag{19}$$

Substituting (15) and (19) into Eqns. (7) and (8) we obtain

$$F'' + \beta(\bar{x})\left(1 - \frac{G}{Ge}\right)\left(1 - F^2\right) + \left(1 - \frac{G}{Ge}\right) f F' + \left(\frac{G'}{Ge - G}\right) F' - PM\,(F-1)\left(1 - \frac{G}{Ge}\right) = 2\left(1 - \frac{G}{Ge}\right) S(\bar{x})\left(F F_{\bar{x}} - F' f_{\bar{x}}\right) \tag{20}$$

$$\mathrm{Pr}^{-1} G'' + f G' = 2S(\bar{x})\left(F G_{\bar{x}} - G' f_{\bar{x}}\right) \tag{21}$$

The boundary conditions (10) reduce to

$$F(\bar{x},0) = 0, \;\; G(\bar{x},0) = 1 \;\; \text{and} \;\; F(\bar{x},\infty) = 1, \;\; G(\bar{x},\infty) = 0 \tag{22}$$

The skin friction coefficient and, the heat transfer coefficient in the form of Nusselt number can be expressed as

$$C_f \left(\mathrm{Re}_L\right)^{1/2} = \left(\frac{G_e}{G_e - 1}\right) 4 P_2 P_1^{1/2} F'_w \;; \qquad Nu\left(\mathrm{Re}_L\right)^{-1/2} = 2^{1/2} \cos\left(\bar{x}/2\right)\left(-G'_w\right) \tag{23}$$

It is worth mentioning here that Kao and Elrod [7] have studied the non similar boundary layer flow, for two dimensional bodies, considering the Eqns. (7) and (8) in the absence of slot suction (injection), magnetic field and temperature-dependent viscosity. Futher, when M $\neq$ 0 the Eqns.(7) and (8) are exactly same as those of Meena and Nath [9], without slot suction (injection) and temperature-dependent viscosity.

METHOD OF SOLUTION

Applying quasilinearization technique [18], we replace the non-linear partial differential equations (6) and (7) by an iterative sequence of linear equations as follows:

$$F''^{(k+1)} + X_1^{(k)} F'^{(k+1)} + X_2^{(k)} F^{(k+1)} + X_3^{(k)} G'^{(k+1)} + X_4^{(k)} G^{(k+1)} + X_5^{(k)} F_\xi^{(k+1)} = U_1^{(k)} \quad (24)$$

$$G''^{(k+1)} + Y_1^{(k)} G'^{(k+1)} + Y_2^{(k)} F^{(k+1)} + Y_3^{(k)} G_\xi^{(k+1)} = U_2^{(k)} \quad (25)$$

where the coefficient functions with iterative index k are known and functions with iterative index k + 1 are to be determined. The boundary conditions become

$$F^{(k+1)}(\xi,0)=0,\ \ G^{(k+1)}(\xi,0)=1\ \text{ and }\ F^{(k+1)}(\xi,\infty)=1,\ \ G^{(k+1)}(\xi,\infty)=0 \quad (26)$$

The coefficients in Eqns.(24) and (25) are given by

$$X_1^{(k)} = \left(1-\frac{G}{Ge}\right)f + \left(\frac{G'}{Ge-G}\right) + \left(1-\frac{G}{Ge}\right)2\xi f_\xi$$

$$X_2^{(k)} = -\beta\left(1-\frac{G}{Ge}\right)(2F) - PM\left(1-\frac{G}{Ge}\right) - 2\xi\left(1-\frac{G}{Ge}\right)F_\xi$$

$$X_3^{(k)} = \frac{F'}{Ge-G}$$

$$X_4^{(k)} = -\beta\left(\frac{1}{Ge}\right)\left(1-F^2\right) - fF'\left(\frac{1}{Ge}\right) + \frac{G'F'}{(Ge-G)^2} + PM(F-1)\left(\frac{1}{Ge}\right) + 2\xi\left(\frac{1}{Ge}\right)\left(FF_\xi - F'f_\xi\right)$$

$$X_5^{(k)} = -2\xi\left(1-\frac{G}{Ge}\right)F$$

$$U_1^{(k)} = -\beta\left(1-\frac{G}{Ge}\right)\left(1+F^2\right) + \frac{G'F'}{Ge-G} - \beta\left(1-F^2\right)\left(\frac{G}{Ge}\right) - fF'\left(\frac{G}{Ge}\right) + \frac{GG'F'}{(Ge-G)^2} + PMF\left(\frac{G}{Ge}\right)$$
$$+ 2\xi\left(\frac{G}{Ge}\right)\left(FF_\xi - F'f_\xi\right) - 2\xi FF_\xi\left(1-\frac{G}{Ge}\right) - PM$$

$$Y_1^{(k)} = \Pr f + 2\xi \Pr f_\xi$$

$$Y_2^{(k)} = -\Pr 2\xi G_\xi$$

$$Y_3^{(k)} = -\Pr 2\xi F$$

$$U_2^{(k)} = -2\xi \Pr F G_\xi$$

The resulting linear partial differential equations (24) and (25) along with (26) were expressed in difference form, considering central difference in $\eta-$direction and backward difference in $\xi-$direction. The equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is solved using Varga's algorithm [19]. In order to obtain grid independent numerical results, the grid sizes $\Delta\eta$ and $\Delta\bar{x}$ have been optimized. To achieve this, the computed values of physical parameters ($F'_w, -G'_w$) with a step size $\Delta\eta$ (keeping $\Delta\bar{x}$ fixed) are compared with those obtained using reduced step size viz., $(\Delta\eta/2), (\Delta\eta/4)$ and so on. Thus optimal values of the step size $\Delta\eta = \Delta\bar{x} = 0.05$ have been used for computation, or for $\bar{x} \le 1.5$. However, for $\bar{x} > 1.5$, finer step size for $\bar{x}$ has been used and in the neighbourhood of the point of zero skin friction $\Delta\bar{x} = 0.0005$ is used. The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 4.0 through out the computation. A convergence criterion based on the relative difference between the current and the previous iteration has been used. The solution is assumed to have converged and the iterative process is terminated when

$$Max\left[\,|(F')_w^{(k+1)} - (F')_w^{(k)}|, |(G')_w^{(k+1)} - (G')_w^{(k)}|\,\right] < 10^{-4}$$

RESULT AND DISCUSSION

Results are obtained for various values of magnetic field M (0.0 ≤ M ≤ 2.0) and temperature-dependent viscosity Ge (1.5 ≤ Ge ≤ 3.0) as per the method of solution discussed in the previous section. In order to validate the accuracy of the numerical method used, our skin friction coefficient and heat transfer coefficient results are compared with those of Kao and Elrod [7] without magnetic field, and Meena and Nath [9] without slot suction(injection) [See Fig.2 (a) and 2(b)]. It is observed that the computed results are in good agreement, with the above studies [7, 9].

In the subsequent paragraph we have discussed the effect of slot injection (suction) on skin friction and heat transfer coefficients with temperature dependent viscosity and in presence of an applied magnetic field.

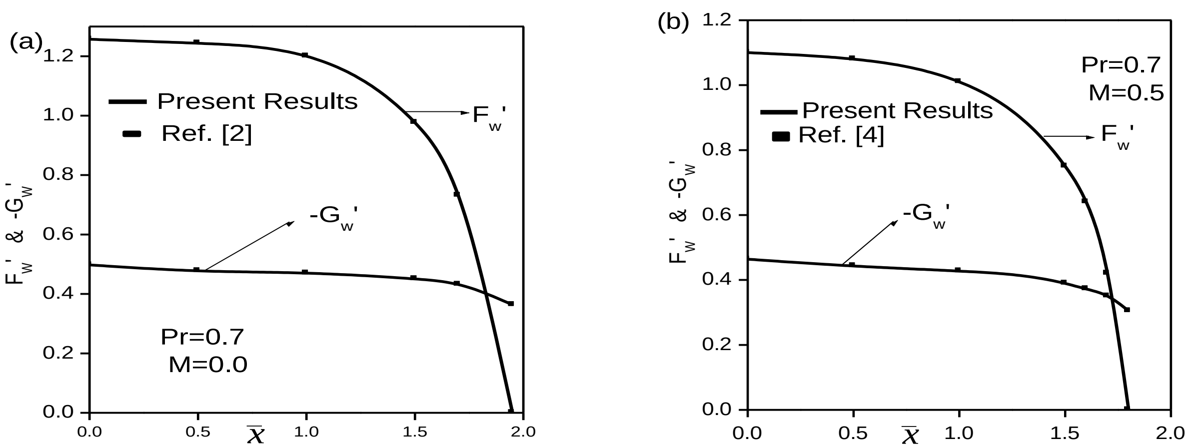

Fig.2 Comparision of skin friction and heat transfer results with those of (a) Kao and Elrod [7] (b) Meena and Nath [9]

Fig. 3 and Fig. 4 depicts the effect of magnetic field (M) and temperature dependent viscosity parameter (Ge) on skin friction coefficient [$C_f\ (Re_L)^{1/2}$] and heat transfer coefficient [$Nu(Re_L)^{-1/2}$], respectively. It is observed that both skin friction and heat transfer coefficient increases as magnetic field and temperature dependent viscosity increases. The percentage of increase of skin friction coefficient is about 336 % and percentage of increase of heat transfer coefficient is about 5.72 % in the range $0.0 \le M \le 2.0$ at an arbitrary value of $\bar{x}$ ($\bar{x} = 1.0$) [see Fig 3(a) & 3(b)]. On the other hand the percentage of increase of skin friction coefficient is about 141 % and percentage of increase of heat transfer coefficient is about 6.22 % in the range $1.5 \le Ge \le 3.0$ at an arbitrary value of $\bar{x}$ ($\bar{x} = 1.0$) [see Fig 4(a) & 4(b)]. Also it is clear from Fig. 3 and Fig. 4 that the boundary layer separation is delayed due to the effect of both magnetic parameter as well as variable viscosity parameter.

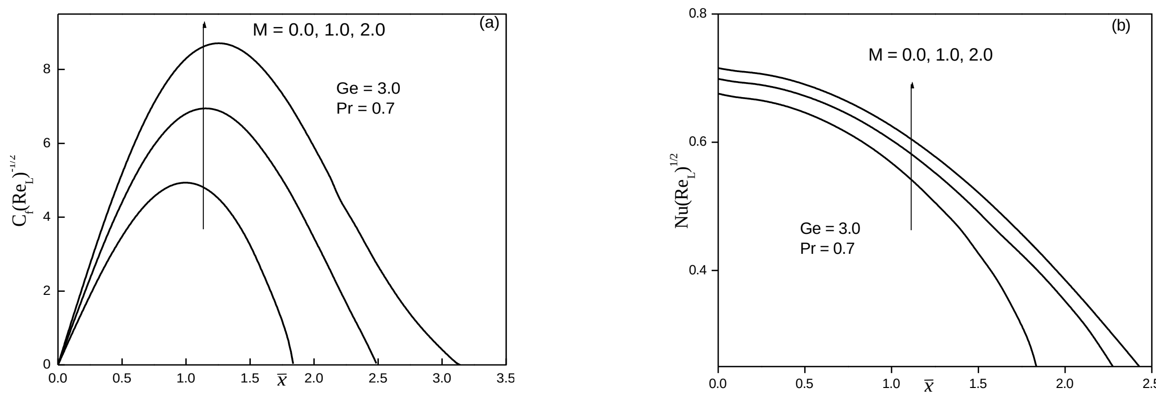

Fig.3. Effect of magnetic field on (a) skin friction coefficient and (b) heat transfer coefficient

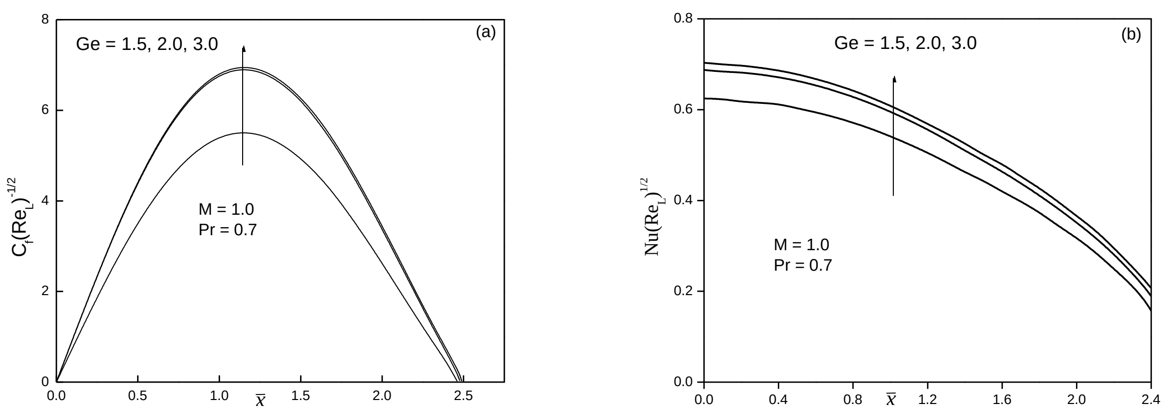

Fig.4. Effect of temperature dependent viscosity on (a) skin friction coefficient and (b) heat transfer coefficient

Fig.5 and Fig.6 show the effect of non-uniform slot suction (or injection) parameter (A>0 or A<0) and $\bar{x}_0$ (which fixes the slot location) on the skin friction [$C_f\,(Re_L)^{1/2}$] and heat transfer coefficients [$Nu(Re_L)^{-1/2}$]. In the case of slot suction (A>0), both $C_f\,(Re_L)^{1/2}$ and $N_u(Re_L)^{-1/2}$ increase and attain their maximum values before the trailing edge of the slot. Further, both skin friction and heat transfer coefficients decrease from their maximum values and $C_f\,(Re_L)^{1/2}$ reaches zero, but $N_u(Re_L)^{-1/2}$ remains finite. It is clear that (See Fig 5) slot suction delays boundary layer separation, but injection through a slot on the body surface has the reverse effect, i.e due to slot injection (A<0), the boundary layer separation occurs in advance and, heat transfer gets reduced inside the slots (See Fig 6). Further, if we move the location of the slot downstream, the point of separation also moves downstream as evident in the Fig. 5.

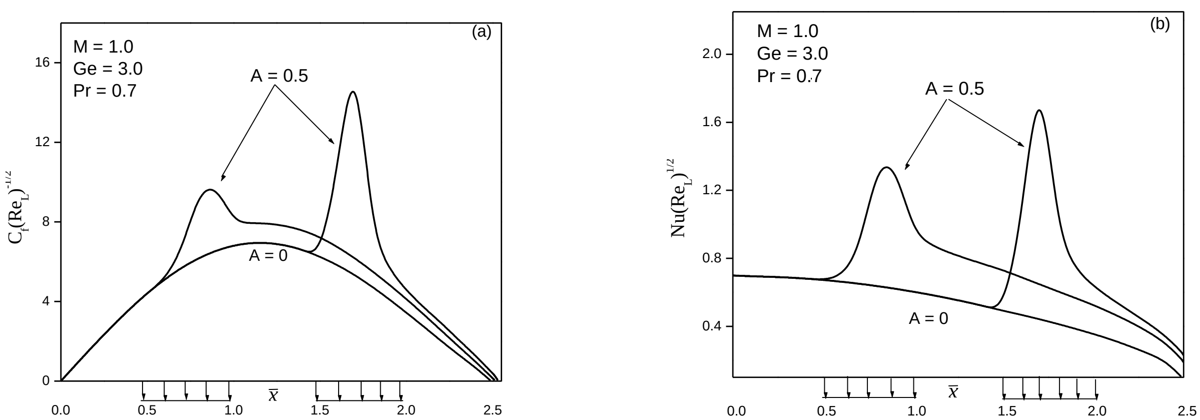

Fig.5. Effect of suction on (a) skin friction coefficient and (b) heat transfer coefficient
$[\bar{x}_{0,}\ \bar{x}_0^*] = [0.5,\ 1.0]\,\&\,[1.5,\ 2.0]$

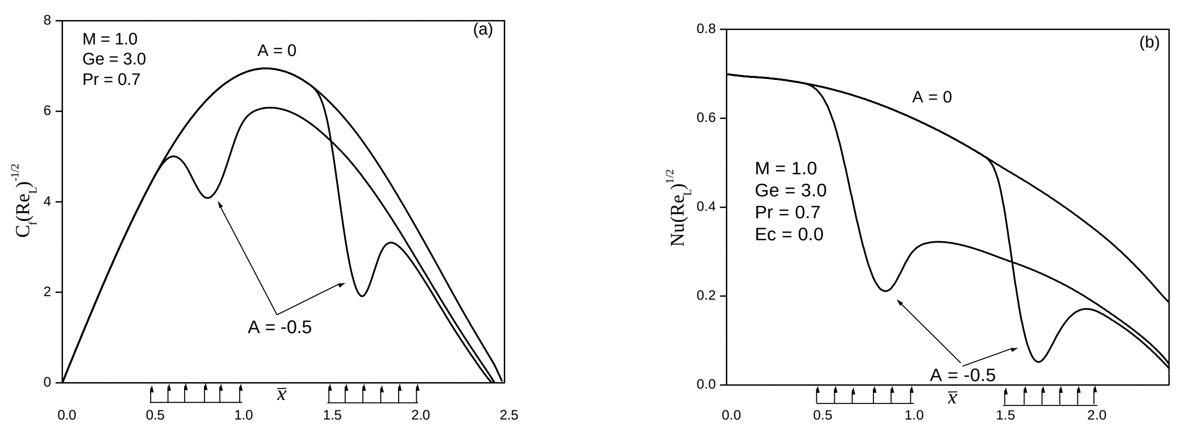

Fig.6. Effect of injection on (a) skin friction coefficient and (b) heat transfer coefficient
$[\bar{x}_{0,}\ \bar{x}_0^*] = [0.5,\ 1.0]\,\&\,[1.5,\ 2.0]$

The velocity and temperature profiles, for different streamwise locations, are obtained for suction (A>0) and injection (A<0) are displayed in Fig.7. It is observed that during suction [Fig. 7(a)] the momentum and the thermal boundary layer thickness decrease, whereas during injection [Fig. 7(b)] they increase.

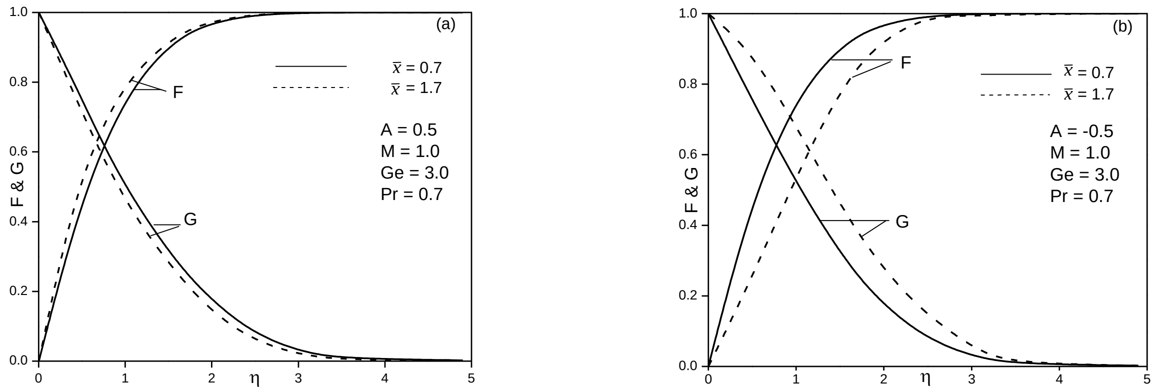

Fig 7. Effect of suction (a) and injection (b) on velocity and temperature profiles.

CONCLUSION

The steady, non-similar two dimensional MHD boundary layer flow and heat transfer in the presence of uniform transverse magnetic field and temperature-dependent viscosity with non-uniform slot suction (injection) has been studied. Numerical solutions have been obtained by a stable implicit finite-difference method along with a quasilinearization technique. From the present results, it is found that the temperature dependent viscosity and magnetic field enhances both skin friction and heat transfer coefficients. Further it is found that the boundary layer separation can be delayed by slot suction and also by moving the slot downstream, whereas slot injection does the opposite.

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