

ELECTRICAL NETWORKS BY GRAPH THEORY

V. Manjula, B E D Dept, MIC College of Technology, Kanchikacherla,
Email manju_adiraju@yahoo.co.in,

ABSTRACT:

The present paper serves an essential and introductory material on graph application to Network analysis. The primary goal of the paper is enable the students have a firm grasp over basic principles of electrical circuits and the ability to design practical circuit that perform the desired operations. Emphasis is placed on basic laws and techniques which are used to develop a working knowledge of methods of analysis used in further topics of electrical engineering. This portion requires some elementary concepts from set theory, matrix algebra and no special background except certain amount of mathematical maturity..Graph theory will be applied in solving practical problems in network analysis, in circuit lay out or in data structures, it leads to large graphs and graphs virtually impossible to analyze with out the aid of computer. In fact high speed digital computer is one of the reason for recent growth of interest in Graph theory This paper shows how electrical network problems can be represented by drawing graphs. Electrical net work analysis and synthesis are mainly the study of network topology. Computer programmers are now available for analysis of large networks based on the graph theoretic approach. For the sake of simplicity in case of networks with active devices the approach of a simple graph was used. The bare essentials of topological analysis of network have been presented in this paper for introducing the student to the application of graph theory in network analysis.

Key words: Introduction ,importance ,representation, Analysis

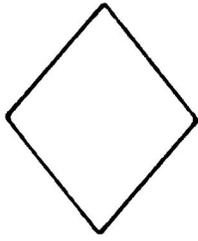
Origin and Growth: In 1736, Swiss mathematician Leonhard Euler presented general theory that included a solution to a problem known as the Königsberg bridge problem, a problem that has ignited the thorough study of graph theory until now.. It contains two islands connected to each other and the river banks having a total of seven bridges The vertices of the graph represent the locations of the map, while the edges of the graph represent the seven Königsberg bridges. The idea of using graph theory for electric network was originated with G .Kirchhoff in 1847 and was improved upon J.C. Maxwell in 1892. A mile stone in Graph theoretical analysis of electrical network was achieved by W.S. Persival

IMPORTANCE: Graphs are being equipped with basic commonsense, Language, analysis and, skill for studying the hardware of any electrical systems. Graph theory is a very interesting topic in discrete mathematics due to its numerous applications especially in the fields of electrical and electronics engineering . It can be applied to different problems such as determining whether a circuit can be implemented in a printed circuit board without the use of jumpers or through-holes for double sided ones and determining the optimum layout of CMOS circuits

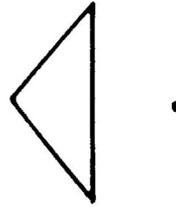
. **WHY Graph Application needed:** In order to describe the geometrical structure of network it is sufficient to replace the network components by single line segments are called elements and their terminals are called nodes. The general problem of network analysis can be stated as follows. Given a network whose structure determines the matrix A ,to find the node voltage inversion matrix can be determined. It requires computation of its determinant and all of its cofactors. These entries consists of polynomials must be carried in literal form. Therefore the usual methods of matrix inversion are computationally difficult to implement .These problems can be circumvented by using graph theory to evaluate determinants and cofactors.

ANALYSIS OF ELECTRICAL NETWORK: The formulation of suitable mathematical model is the first step in the analysis of electrical network. The model must describe the characteristics of individual network components as well as the relations that govern the interaction of these elements. The electrical characteristics of individual network components can be presented conveniently in the form of primitive network matrix that describes the performance of inter connected network.

GRAPH THEORY INTRODUCTION: A graph basically is a pictorial representation of a system using two basic elements nodes and edges. A **node** or **vertex** is represented by a, and an **edge** is represented by a line segment connecting two nodes. The graph is called an **undirected graph** because all of its edges do not indicate any direction from one node to the other. However, if the edges of a graph direct one node to another node, , then the graph is said to be a **directed graph**. A graph G can be mathematically represented by a double (V,E) , where V is the set of all vertices, and E is the set of all edges. If the graph $G(V,E)$ is undirected, then each edge e in E is associated with the vertices v and w , and is written as either $e = \{v,w\}$ or $e = \{w,v\}$, i.e. the edge e connects vertices v and w . However, if the graph $G(V,E)$ is a directed graph, then each edge e in E is associated with an *ordered pair* of vertices v and w , and is uniquely written as $e = (v, w)$, i.e. the edge connects vertex v to vertex w ,. The association of an edge e to a pair of vertices, for example v and w , means that e is **incident on** v and w , and v and w are said to be **incident on** e . Vertices v and w , on the other hand, are **adjacent**. A, in this lecture note, it is assumed that the number of vertices and If the value is numerical, they are called **weighted graphs** In a graph, two or more edges are called **parallel edges** if all of these edges are associated with the same pair of vertices v and w . If an edge, on the other hand, is incident on a single vertex, then that edge is called a **loop**



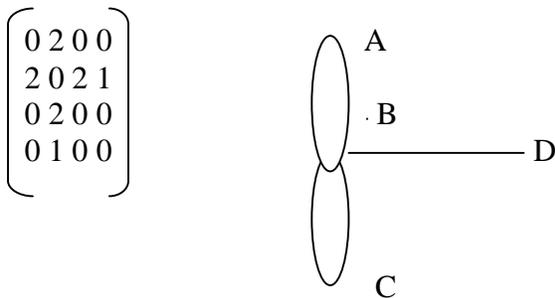
Connected graph



Disconnected graph

GRAPH REPRESENTATION:

Graphs can actually be represented using matrices. Two of the most widely used matrices for graph representation are adjacency matrices and incidence matrices. An **adjacency matrix** is a square matrix in which each row and column is represented by a vertex, $V = \{A, B, C, D\}$. This means that the square matrix must be 4×4 , i.e. each row and column is represented by each of the four vertices in V . Thus, following is the the matrix representation



TREE and CUT TREE: A tree is a connected sub graph of network which consists of all the nodes of original graph but no closed paths. The graph of network may have a number of trees. The number of nodes in the graph is equal to number of nodes in the tree.

Parallel Circuits & KCL in NETWORKS

Kirchhoff's Voltage Law states that for a closed loop:

$$\sum V = 0, \text{ or } \sum \text{V rises} = \sum \text{V drops}$$

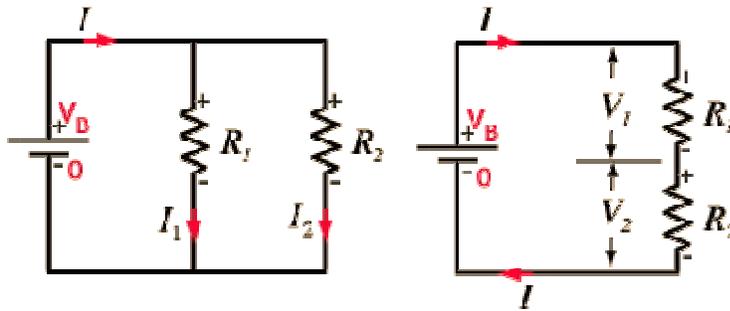
The total resistance of n resistors in series is:

$$R_T = R_1 + R_2 + \dots + R_n$$

The total power is: $P_T = P_1 + P_2 + \dots + P_n$

IN SERIES, so that the same current flows through all the components but a different potential difference (voltage) can exist across each one.

IN PARALLEL, so that the same potential difference (voltage) exists across all the components but each component may carry a different current



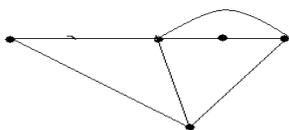
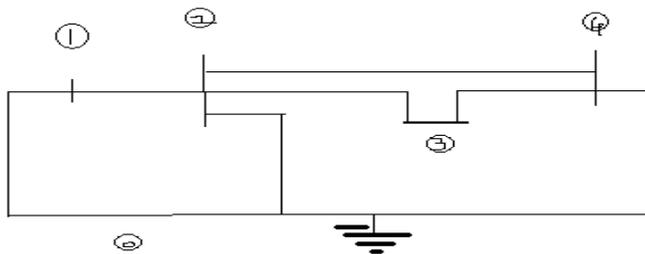
Parallel resistors

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2}$$

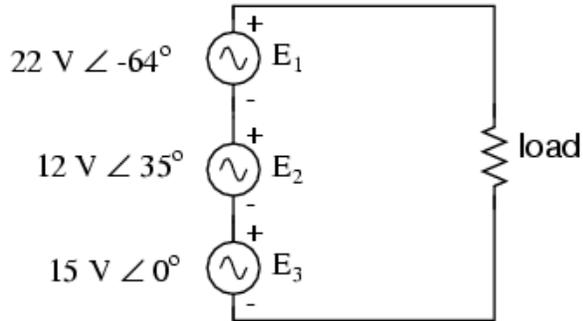
Series resistors

$$R_{equivalent} = R_1 + R_2$$

In electrical engineering the term branch is used for edge, node for vertex, and loop for circuit. An electrical network is the collection of interconnected electric elements such as resistors, capacitors, Inductors etc. The behavior is a function of the factors the characteristics of each of the elements, how they are connected together. A positive sequence network diagram and its connected graph are as follows.



The three AC voltage sources in series and use complex numbers to determine additive voltages. All the rules and laws learned in the study of DC circuits are applied to AC circuits as well.



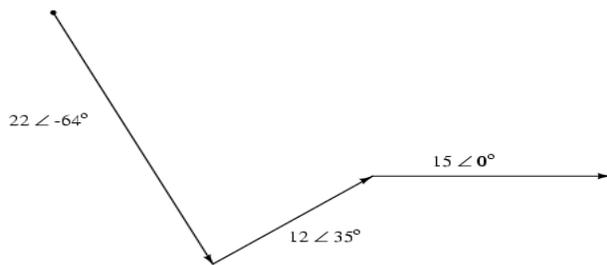
KVL allows addition of complex voltages

The polarity marks for all three voltage sources are oriented in such a way that their stated voltages should add to make the total voltage across the load resistor.

$$E_{\text{total}} = E_1 + E_2 + E_3$$

$$E_{\text{total}} = (22\text{ V} \angle -64^\circ) + (12\text{ V} \angle 35^\circ) + (15\text{ V} \angle 0^\circ)$$

Graphically, the vectors add up as shown in Figure



Graphic addition of vector voltages.

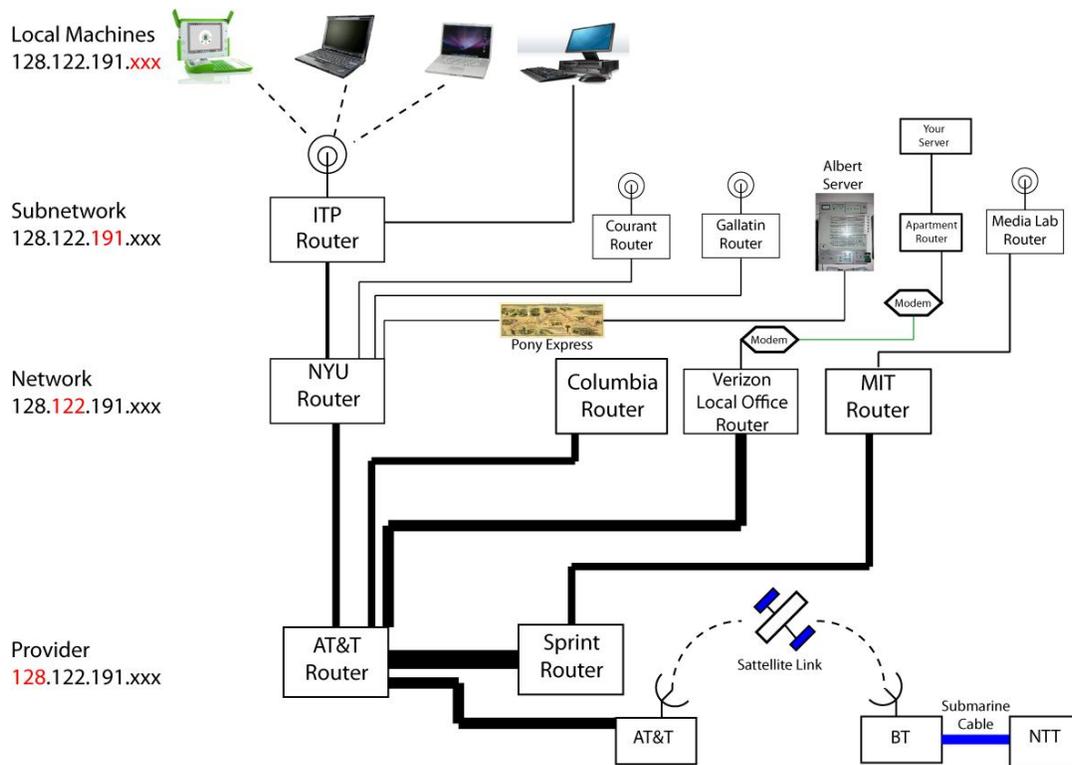
Networks are often not random—density (the average number of links a node has) is not a Poisson distribution, but generally follows a Power Law distribution

Poisson Distribution (similar to Normal curve, representing a random number of links per node

Power Law Distribution



Graphical representations of a network neatly explain the interconnectedness of nodes, and makes it possible to understand the relationship between them.



Maximum hops can be determined by counting the steps, often requiring several iterations or more on more complex networks.

CONCLUSION: Thus the general problem of network analysis can be solved by using graph theory to evaluate the determination and co-factors. Thus the electrical network problems like most linear system problems consist of matrix inversions show how graph theory can be used rather than algebra. Thus real life problems that can be modeled in graphs are small enough to be solved by means of other than graph theory. Thus respect of graph theory is different from college algebra, elementary calculus of its inherent simplicity, graph theory has a very wide range of applications in engineering physical, social and biological sciences. Thus graph theory has more substantive applications particular in solving electrical network problems.

REFERENCES

- [1] Computer methods in power systems by Glenn W. Stagg-Mc Graw –Hill services
- [2] Electrical circuit Analysis by A. Sudhakaran Tata Mc Graw –Hill pvt ltd
- [3] Graph Theory and its Applications to Computer science- Narasih DEO
- [4] Introductory Graph Theory for Electrical and Electronics Engineers
 IEEE MULTIDISCIPLINARY ENGINEERING EDUCATION MAGAZINE,
- [5] **EE110300 Practice of Electrical and Computer Engineering Lecture 2 and Lecture 4.1**
- [6] http://openbookproject.net/electricCircuits/AC/AC_2.html