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WAVE PROPAGATION IN MICRO-ISOTROPIC, MICRO-ELASTIC SOLID WHEN MACRO-ROTATION AND MICRO-ROTATION ARE ZERO:

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ABSTRACT: Wave propagation in micro-isotropic, micro-elastic solid in case of macro-rotation and micro-rotation are zero. It is observed that there are three irrotational waves are propagate.

Key Words: Waves in Micro – isotropic, Micro-elastic solids, zero macro-rotation and Micro-rotation.

INTRODUCTION

The number of problems were studied in the micro polar theory discussed by Eringen and Suhubi [1,2]. However, very few problems were solved by taking the principle of micromorphic theory. This may be due to the fact that equations of micromaphic elasticity are more complicated. If one has to consider a problem where micro-motion is not restricted to rotation only, the problem can be considered by micro-morphic theory. Addressing himself to this more general problem but searching for a more tractable formulation Koh [3] developed a theory by extending the concept of coincidence of principal directions of stress and strain in classical elasticity to the micro-elastic medium through the postulate of principle of coincidence. Imposing a particular form of micro – isotropy, Koh [3] obtained special constraints on the elastic moduli there by reducing the number 18 to 10 in this special case. With the above restrictions on the micro-morphic medium, Koh [3] called this new medium as micro-isotropic, micro-elastic medium.

The action of sudden disturbance in the elastic solid medium is transmitted at once to other parts of the body. At the beginning the remote parts of the body remain undistursed and the deformation produced at a point are propagated through the body in the form of waves. If a region is large then the effects of the boundaries can be dis regarded, it is possible to represent the disturbance as the sum of two waves propagated with velocities that depend only the density and elastic constants of the medium. Indeed, the displacement vector $\vec{u}$ can be represented as a sum of two vectors one of which is solenoidal and other is irrotational. This leads to a consideration of two special types of disturbance for one of which div $\vec{u} = 0$ and for the other curl $\vec{u} = \vec{0}$ in the case of classical elasticity [4]. In this paper, we study the wave propagation in micro-isotropic, micro-elastic solid in case of zero macro-rotation $\vec{r}$ and zero micro-rotation $\vec{\phi}$.

BASIC EQUATIONS

The basic equations for a micro- isotropic, micro elastic solids are obtained by Koh [3].

The constitutive equations are:

$$t_{(km)} = A_1 e_{pp} \delta_{km} + 2A_2 e_{km} \quad \ldots\ldots (1)$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \varepsilon_{pkm} (r_p + \phi_p) \quad \ldots\ldots (2)$$

$$\sigma_{(km)} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)} \quad \ldots\ldots (3)$$

$$t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(mn),k} \quad \ldots\ldots (4)$$

$$m_{kl} = -2(B_5 \phi_{l,k} + B_4 \phi_{k,l} + B_5 \phi_{p,p} \delta_{kl}) \quad \ldots\ldots (5)$$

where ( ) denotes symmetric part and [ ] denotes anti – symmetric part and

$$A_1 = \lambda + \sigma_1 ; \quad B_1 = \tau_3$$
$$A_2 = \mu + \sigma_2 ; \quad 2B_2 = \tau_7 + \tau_{10}$$
$$A_3 = \sigma_5 ; \quad B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}$$
$$A_4 = -\sigma_1 ; \quad B_4 = -2\tau_4 \quad \ldots\ldots (6)$$
$$A_5 = -\sigma_2 ; \quad B_5 = -2\tau_9$$

The equations of motion for this material are

$$(A_1 + A_2 - A_3) u_{p,pm} + (A_2 + A_3) u_{m,pp} + 2A_3 \varepsilon_{pkm} \phi_{p,k} + \rho fm = \rho \frac{\partial^2 u_m}{\partial t^2} \quad \ldots\ldots (7)$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j\frac{\partial^2\phi_{(ij)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j\frac{\partial^2\phi_p}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where $\phi_p = \frac{1}{2}\varepsilon_{pkm}\phi_{km}$; $r_p = \frac{1}{2}\varepsilon_{pkm}u_{m,k}$;

couple stress tensor $m_{kp} = \varepsilon_{pnm}t_{kmn}$;

body couples $l_p = \varepsilon_{pnm}f_{mn}$

and ε_{pnm} is permutation symbol.

S. parameshwaran and S.L. Koh [5] establish the following constrains on micromorphic elastic constants c

$$3A_1 + 2A_2 > 0;\ \ A_2 > 0;\ A_3 > 0$$

$$3A_4 + 2A_5 > 0;\ A_5 > 0$$

$$3B_1 + 2B_2 > 0;\ B_2 > 0;\ \ B_3 > 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$-B_3 < B_4 < B_3;\ \ B_3 + B_4 + B_5 > 0$$

EQUATIONS OF MOTION:

We write the equations of motion (7)and (9) in terms of

$$\Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

And $$\Delta^1 = \frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2} + \frac{\partial \phi_3}{\partial x_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where we call Δ is dilation and Δ^1 is microrotation dilation. The equation (7) in components under the absence of body forces are

$$\left(A_1 + A_2 - A_3\right) \frac{\partial}{\partial x_1}\Delta + \left(A_2 + A_3\right)\nabla^2 u_1 + 2A_3\left(\frac{\partial \phi_2}{\partial x_3} - \frac{\partial \phi_3}{\partial x_2}\right) = \rho\frac{\partial^2 u_1}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\left(A_1 + A_2 - A_3\right) \frac{\partial}{\partial x_2}\Delta + \left(A_2 + A_3\right)\nabla^2 u_2 + 2A_3\left(\frac{\partial \phi_3}{\partial x_1} - \frac{\partial \phi_1}{\partial x_3}\right) = \rho\frac{\partial^2 u_2}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\left(A_1 + A_2 - A_3\right) \frac{\partial}{\partial x_3}\Delta + \left(A_2 + A_3\right)\nabla^2 u_3 + 2A_3\left(\frac{\partial \phi_1}{\partial x_2} - \frac{\partial \phi_2}{\partial x_1}\right) = \rho\frac{\partial^2 u_3}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$

The equation (9) in components form under the absence of body couples are

$$2B_3\nabla^2\phi_1 + 2\left(B_4 + B_5\right)\frac{\partial}{\partial x_1}\Delta^1 - 2A_3\left(u_{3,2} - u_{2,3}\right) - 4A_3\phi_1 = \rho j\frac{\partial^2 \phi_1}{\partial t^2} \quad \ldots\ldots\ldots\ldots \quad (16)$$

$$2B_3\nabla^2\phi_2 + 2\left(B_4 + B_5\right)\frac{\partial}{\partial x_2}\Delta^1 - 2A_3\left(u_{1,3} - u_{3,1}\right) - 4A_3\phi_2 = \rho j\frac{\partial^2 \phi_2}{\partial t^2} \quad \ldots\ldots\ldots \quad (17)$$

$$2B_3\nabla^2\phi_3 + 2\left(B_4 + B_5\right)\frac{\partial}{\partial x_3}\Delta^1 - 2A_3\left(u_{2,1} - u_{1,2}\right) - 4A_3\phi_3 = \rho j\frac{\partial^2 \phi_3}{\partial t^2} \quad \ldots\ldots\ldots \quad (18)$$

Now the equations (8) are given by

$$B_1\phi_{pp,kk} + 2B_2\phi_{11,kk} - A_4\phi_{pp} - 2A_5\phi_{11} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{11}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{22,kk} - A_4\phi_{pp} - 2A_5\phi_{22} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{22}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{33,kk} - A_4\phi_{pp} - 2A_5\phi_{33} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{33}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

$$2B_2\phi_{(12),kk} - 2A_5\phi_{(12)} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{(12)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$2B_2\phi_{(13),kk} - 2A_5\phi_{(13)} = \rho j\frac{\partial^2 \phi_{(13)}}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$2B_2\phi_{(23),kk} - 2A_5\phi_{(23)} = \rho j \frac{\partial^2 \phi_{(23)}}{\partial t^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (24)$$

Adding equations (19) to (21) we get

$$(3B_1 + 2B_2)\phi_{pp,kk} - (3A_4 + 2A_5)\phi_{pp} = \frac{1}{2}\rho j \frac{\partial^2 \phi_{pp}}{\partial t^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (25)$$

Subtracting equation (20) from (19) we get the following equation.

$$2B_2(\phi_{11} - \phi_{22})_{kk} - 2A_5(\phi_{11} - \phi_{22}) = \frac{\rho j}{2}\frac{\partial^2}{dt^2}(\phi_{11} - \phi_{22}) \qquad \ldots\ldots\ldots\ldots\ldots \qquad (26)$$

And subtracting equation (21) from (19) we get

$$2B_2(\phi_{11} - \phi_{33}),_{kk} - 2A_5(\phi_{11} - \phi_{33}) = \frac{\rho j}{2}\frac{\partial^2}{dt^2}(\phi_{11} - \phi_{33}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (27)$$

Wave propagation in the unbounded micro-isotropic medium is represented by twelve equations (13) to (18) and (22) to (27).

Propagation of waves when macro-rotation $\vec{r} = \vec{0}$ and Micro-rotation $\vec{\phi} = \vec{0}$

We consider the wave propagation in micro-isotropic, micro-elastic solid in which macro-rotation and micro-rotation are zero, but the deformation consists of macro-strain $\vec{U}$ and micro-strain $\phi_{(ij)}; i, j = 1,2,3$.

Then $\nabla \times \vec{U} = \vec{o}$ gives

$$\frac{\partial u_3}{\partial x_2} = \frac{\partial u_2}{dx_3}$$

$$\frac{\partial u_1}{\partial x_3} = \frac{\partial u_3}{\partial x_1}; \frac{\partial u_2}{\partial x_1} = \frac{\partial u_1}{\partial x_2} \qquad \ldots\ldots (28)$$

and $\vec{\phi} = \vec{0}$ gives

$$\phi_1 = \phi_2 = \phi_3 = 0 \qquad \ldots\ldots(29)$$

where $\vec{U} = u_1 \vec{i}_1 + u_2 \vec{i}_2 + u_3 \vec{i}_3$ and $\vec{\phi} = \phi_1 \vec{i}_1 + \phi_2 \vec{i}_2 + \phi_3 \vec{i}_3$

These conditions are satisfied if the displacements u_1, u_2, u_3 are derivable and

$$\frac{\partial \Delta}{\partial x_1} = \nabla^2 u_1 ; \frac{\partial \Delta}{dx_2} = \nabla^2 u_2 ; \frac{\partial \Delta}{dx_3} = \nabla^2 u_3$$

In this case, the equations (13) to (15) reduce to respectively

$$\left(A_2 + 2A_2\right)\nabla^2 u_1 = \rho \frac{\partial^2 u_1}{\partial t^2} \tag{30}$$

$$\left(A_2 + 2A_2\right)\nabla^2 u_2 = \rho \frac{\partial^2 u_2}{\partial t^2} \tag{31}$$

$$\left(A_2 + 2A_2\right)\nabla^2 u_3 = \rho \frac{\partial^2 u_3}{\partial t^2} \tag{32}$$

The above equations are expressed as

$$\nabla^2 u_1 = \frac{1}{c_1^2} \frac{\partial^2 u_1}{\partial t^2} \tag{33}$$

$$\nabla^2 u_2 = \frac{1}{c_1^2} \frac{\partial^2 u_2}{\partial t^2} \tag{34}$$

$$\nabla^2 u_3 = \frac{1}{c_1^2} \frac{\partial^2 u_3}{\partial t^2} \tag{35}$$

where $c_1^2 = \dfrac{A_1 + 2A_2}{\rho}$

Since the motion involved in this case is irrotational then the equations (33), (34), (35) are known as irrotational waves. The equations (16) to (18) are identically satisfies.

The equations involving micro-strains represent from (22) to (27) and these six waves travel with two distinct velocities having cut-off frequencies without discussed here.

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