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FREE CONVECTION EFFECTS ON MHD STOKES PROBLEM USING FINITE ELEMENT METHOD

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ABSTRACT

A finite element technique is implemented for the free convection effects on MHD stokes problem. The numerical values for velocity and temperature under various physical situations are obtained and have been analyzed graphically. The numerical results are compared with analytical solution and established the validity of finite element technique (Ritz method).

Key words: specific heat, Grashof number, velocity, kinematic Viscosity, Finite element method.

1. Introduction

One of the solutions of the Navier-Stokes equation was first given by Stokes [8] for the case of the flow of an incompressible viscous fluid past an infinite horizontal plate moving impulsively in its own plane. Hence it is known as Stokes first problem. In recent years, the effects of the transverse magnetic field on the flow of an incompressible, viscous, electrically conducting fluid, have also been studied extensively by many researchers. The magneto hydrodynamic aspect of Stokes problem, on neglecting the induced magnetic field, was first presented by Rossow in case of a horizontal plate. How does a transversely applied magnetic field affect the flow of an electrically conducting, viscous incompressible fluid past an impulsively started vertical plate? This indeed had been the motivation for Soundalgekar [7], who had presented an exact of analysis of MHD Stokes problem for the flow of an electrically conducting, incompressible, viscous fluid past an impulsively started vertical plate, under the action of transversely applied magnetic field.

In many industrial applications particularly in the design of space ship, solar energy collectors etc, the flow past an infinite vertical plate, started impulsively from rest, plays an important role.

The flow past a vertical plate moving impulsively in its own plane was studied by Soundalgekar. Further, Georgantopolus and Singh [6] have extended this problem in hydro magnetic, when the magnetic field is fixed relative to the fluid and the plate, respectively. The problem of free convection of an electrically conducting fluid past a plate under the influence of a magnetic field is important and useful partly for gaining basic understanding of such flows, and partly for possible application to geophysical, astrophysical, and aero dynamical problems. In this chapter the effect of an applied magnetic field on an unsteady hydro magnetic flow induced by an infinite vertical moving plate, when the heat flux at the plate is constant is studied.

We have used the finite element method to solve the governing equations when $P_r<1$ and $P_r>1$. In order to assess the accuracy of our method, our numerical solutions for $P_r = 1$ have been compared with the analytical solutions. The results have been discussed under section 5.

2. Mathematical Formulation

We consider the flow of an electrically conducting, incompressible, viscous fluid past an impulsively started infinite vertical plate, moving in its own plate. The x_1-axis is taken along the vertical plate in the direction of motion and the y_1-axis is taken perpendicular to it.

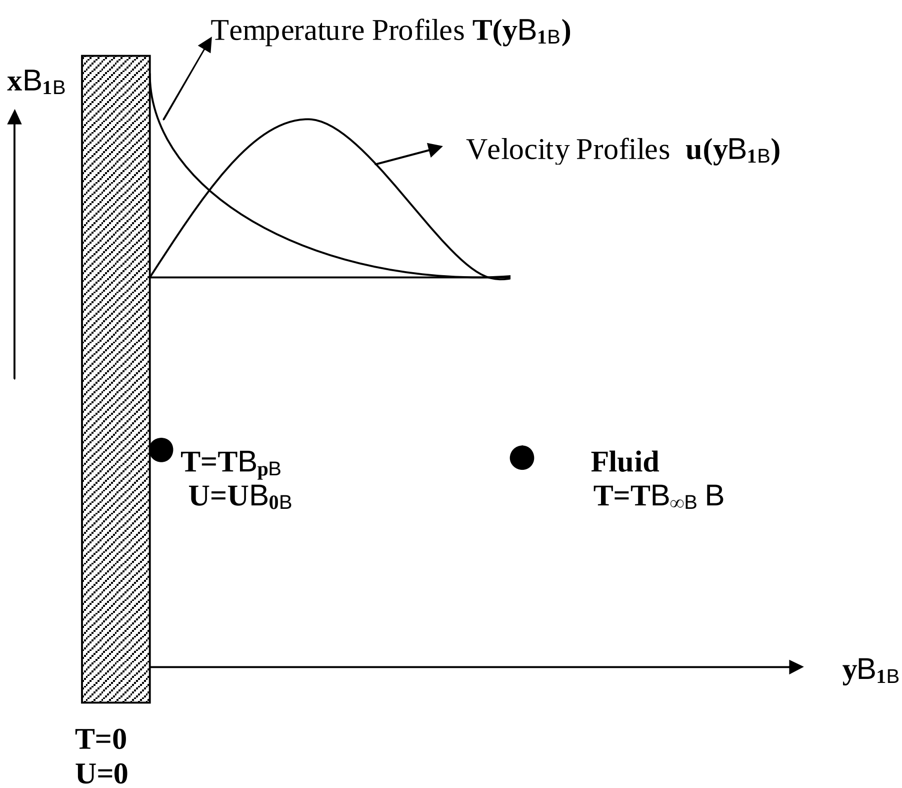

Fig.1. Geometry of the problem

It is assumed that the magnetic field lines are perpendicular to the free-stream velocity and the magnetic permeability μ_e is constant throughout the field. Moreover, the induced magnetic field, produced by the motion of electrically conducting fluid is assumed to be negligible. Hence, the components of electromagnetic induction are

$$B_{x_1} = 0, \quad B_{y_1} = \mu_e H_0 = B_0(constant) \quad \text{and} \quad B_{z_1} = 0. \qquad (1)$$

As the plate is infinite, all variables in the problem are functions of y_1 and t_1 only. Therefore, the components of velocity are given by

$$v_{x_1} = u_1(y_1, t_1)\ , \qquad v_{y_1} = 0 \qquad \text{and} \qquad v_{z_1} = 0. \qquad (2)$$

If we set the electric field E=0(as no applied or polarization voltage exists) the Lorentz force has components

$$F_{x_1} = -\frac{\sigma}{\rho} B_0{}^2 u_1, \quad F_{y_1} = 0 \quad \text{and} \quad F_{z_1} = 0. \tag{3}$$

Within the framework of these assumptions, the equations which govern the free-convective flow of an electrically conducting fluid under Boussinesq's approximation, are given by

$$\frac{\partial u_1}{\partial t_1} = g\beta\left(T_1 - T_\infty\right) + \nu \frac{\partial^2 u_1}{\partial y_1^2} - \frac{\sigma B_0^2}{\rho_1} u_1, \tag{4}$$

$$\rho_1 c_p \frac{\partial T_1}{\partial t_1} = k \frac{\partial^2 T_1}{\partial y_1^2}. \tag{5}$$

Here, u_1 is the velocity of the fluid, $T_1$the temperature of the fluid near the plate, T_∞ the temperature of the fluid far away from the plate, g the acceleration due to gravity, β the coefficient of volume expansion, ν the kinematics viscosity, σ the scalar electrical conductivity, B_0 the applied uniform magnetic field, ρ_1 the density of the fluid, k the thermal conductivity and t_1 is the time.

In equation (5) the heat due to viscous dissipation is assumed to be negligible. This is possible when the velocity is small. The boundary conditions are

$$\left.\begin{aligned} &u_1(y_1, t_1) = U_0, \\ &T_1(y_1, t_1) = T_p \qquad \text{at } y_1 = 0; \\ &u_1(y_1, t_1) \to 0, \\ &T_1(y_1, t_1) \to T_\infty \qquad \text{as } y_1 \to \infty. \end{aligned}\right\} \tag{6}$$

On introducing the following non-dimensional quantities

$$\left.\begin{aligned} &y = \frac{y_1 U_0}{\nu}, \\ &t = \frac{t_1 U_0^2}{\nu}, \\ &u = \frac{u_1}{U_0}, \\ &T = \frac{T_1 - T_\infty}{T_p - T_\infty}, \\ &P_r = \frac{\mu c_p}{k} \text{ (Prandtl number)}, \\ &G_r = \frac{\nu g\beta\left(T_P - T_\infty\right)}{U_0^3} \text{ (Grashof number)}, \\ &M^2 = \frac{\sigma B_0^2 \nu}{\rho_1 U_0^2} \text{ (magnetic field parameter)}. \end{aligned}\right\} \tag{7}$$

in equations (4), (5) and (6), we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r T - M^2 u, \qquad (8)$$

$$P_r \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2}, \qquad (9)$$

With

$$\left.\begin{array}{ll} u(0,t) = 1, & \\ T(0,t) = 1, & \text{at } y = 0, \\ u(\infty,t) = 0, & \\ T(\infty,t) = 0, & \text{as } y \to \infty. \end{array}\right\} \qquad (10)$$

3. Method of solution

The Finite Element Technique (Ritz method) is used to solve equations (8) and (9) subject to (10).

Now, we consider equation (9)

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2}.$$

Using finite element method with crank-nicolson discretization taking h=0.05, k=0.0025 therefore $r = k / h^2 = 1$.

The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem may be written as

$$j^e = 1/2 \int_{yj}^{yk} \left[\left(\frac{\partial T^e}{\partial y} \right)^2 + 2T^e p_r \frac{\partial T^e}{\partial t} \right] dy$$

For the linear piecewise approximate solution

$$T^e = N_J(y) T_j(t) + N_k(y) T_k(t)$$

The element equation is given by

$$j^e = 1/2 \int_{y_j}^{y_k} \left\{ \left([N_j^{'} \quad N_k^{'}] \begin{bmatrix} T_J \\ T_k \end{bmatrix} \right)^2 + 2P_r [N_J \quad N_K] \begin{bmatrix} T_J \\ T_k \end{bmatrix} [N_j \quad N_K] \begin{bmatrix} \dot{T}_j \\ \dot{T}_k \end{bmatrix} \right\} dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k} \left\{ \begin{pmatrix} [N_j^{'} N_j^{'} & N_j^{'} N_k^{'}] \\ [N_k^{'} N_j^{'} & N_k^{'} N_k^{'}] \end{pmatrix} \begin{bmatrix} T_j \\ T_k \end{bmatrix} + P_r \begin{pmatrix} [N_j N_j & N_j N_k] \\ [N_k N_j & N_k N_k] \end{pmatrix} \begin{bmatrix} \dot{T}_j \\ \dot{T}_k \end{bmatrix} \right\} dy$$

Where prime denotes differentiation w.r.t. to ‘y’ and dot represent differentiation w.r.t. to ‘t’

Here $N_j^{'}=\frac{-1}{h}$; $N_k^{'}=\frac{1}{h}$; where h= $y_k - y_j$;

Simplifying above equation we get

$$\left(1-\frac{3r}{p_r}\right)T_{i-1}^{n+1}+\left(4+\frac{6r}{p_r}\right)T_i^{n+1}+\left(1-\frac{3r}{p_r}\right)T_{i+1}^{n+1}=$$
$$\left(1+\frac{3r}{p_r}\right)T_{i-1}^{n}+\left(4-\frac{6r}{p_r}\right)T_{i-1}^{n}+\left(1+\frac{3r}{p_r}\right)T_{i-1}^{n}$$

h=0.05, k=0.0025 and r=1 the nodal points (y_n, t_n) are shown in the figure (2)
Where i=1(1)9 and n=1, 2, 3….
We solve the system of above equations using Gauss –Seidel method

4. Numerical Solutions and their accuracy

To get the numerical solutions of temperature T we have taken the aid of the computer by developing a code (program) in C language. It is clear from the figure (2) the computed results are very close to analytical values (table 1) hence, the finite element technique is valid.

Now, we consider equation (8)

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+G_r T-M^2 u$$

The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem may be written as

$$j^e = 1/2\int_{yj}^{yk}\left[\left(\frac{\partial U^e}{\partial y}\right)^2+M^2U^{e2}+2U^e\frac{\partial U^e}{\partial t}-2U^eG_rT\right]dy$$

Simplifying above equation we get

$$(1\text{-}3r+\frac{M^2rh^2}{2})U_{i-1}^{n+1}+(4+6r+2rM^2h^2)\,U_i^{n+1}+(1\text{-}3r+\frac{M^2rh^2}{2})U_{i+1}^{n+1}=$$
$$(1+3r-\frac{M^2rh^2}{2})U_{i-1}^{n}+(4\text{-}6r\text{-}2rM^2h^2)\,U_{i-1}^{n}+(1+3r-\frac{M^2rh^2}{2})\,U_{i-1}^{n}+6\,G_rT_i^n\,k$$

We solve the system of above equations using Gauss –Seidel method.
To get the numerical solutions of velocity U we have taken the aid of the computer by developing a code (program) in C language. The numerical and analytical values for different t and y are presented in table (2).

5. Discussion:

Numerical solutions with analytical solutions are plotted in figure (3) using table (2). The numerical solution very close to analytical solution. Hence, this shows the validity of the method.

Computer program (code in C language) for different values of prandtl number P_r<1 and P_r=1 and different values for M and Grashof number G_r and different and fixed time level are computed and shown graphically

6. Results and discussion

To gain physical insight into the problem, numerical values are obtained and displayed in graphical form. The temperature profiles are shown in Figures (4) and (5), respectively, for P_r<1 and P_r=1. An increase in P_r leads to a decrease in temperature for fixed values of t. Further, as time increases, the temperature also increases when P_r is kept constant.

The velocity profiles in presence of magnetic field are displayed in figures (6) to (9). All real values for G_r are chosen as they are interesting from the physical point of view. Now free convection currents exist because of the $P_r(T_p-T_\infty)$ which may be positive, zero or negative. As we know the Grashof number G_r is a common dimensionless group that is used when analyzing the potential effect of convection introduced by large temperature differences. So G_r will assume positive, zero or negative values. From the physical point of view, $G_r < 0$ corresponds to an externally heated plate as free convection currents are carried towards the plate. $G_r > 0$ corresponds to an externally cooled plate and $G_r = 0$ corresponds to the absence of free convection currents.

In figure (6) velocity profiles are shown for P_r<1 in the case when the plate is heated by the free convection currents. An increase in P_r leads to an increase in velocity when the values of G_r, t and M are kept fixed. An increase in M leads to a decrease in the velocity. As time is increased, the velocity decreases when the values of G_r, M and P_r are kept fixed. Greater heating of the plate by the free convection currents also cause an increase in the velocity.

In figure (8) velocity profiles are shown in the case of the plate being cooled by the free convection currents. Here an increase in P_r leads to a decrease in the velocity when the values of G_r, t, M are kept constant. An increase in M or t leads to a decrease in the velocity but greater cooling of the plate by free convection currents cause a rise in the velocity.

In figure (7) and (9), the velocity profiles are shown for fluids whose Prandtl number is unity.

A detailed numerical study has been carried out for an MHD Stokes problem for an infinite vertical plate. The main conclusions of this study are as follows:

(i) An increase in ` $P_r$ ' leads to decrease in temperature when the values of `t’ are kept constant. Also, also time increases, the temperature also increases when ` P_r ' is kept fixed.

(ii) Greater heating on of the plate, the velocity increases, whereas on greater cooling of the plate the velocity decreases.

	Y	Analytical	Numerical		

Table1. Temperature Comparison of profiles when P_r=0.7333

t		Solution	Solution	Percentage of Error |
0.0075	0	1	1	0.0000
0.0075	0.05	0.7261	0.6799	0.0005
0.0075	0.1	0.4846	0.4664	0.0002
0.0075	0.15	0.2944	0.2502	0.0004
0.0075	0.2	0.1621	0.1144	0.0005
0.0075	0.25	0.0805	0.0472	0.0003
0.0075	0.3	0.0360	0.0182	0.0002
0.0075	0.35	0.0144	0.0067	0.0001
0.0075	0.4	0.0052	0.0023	0.0000
0.0075	0.45	0.0017	0.0008	0.0000
0.0075	0.5	0.0005	0.0003	0.0000
0.0075	0.55	0.0001	0.0001	0.0000
0.0075	0.6	0.0000	0.0000	0.0000
0.0075	0.65	0.0000	0.0000	0.0000
0.0075	0.7	0.0000	0.0000	0.0000
0.0075	0.75	0.0000	0.0000	0.0000
0.0075	0.8	0.0000	0.0000	0.0000
0.0075	0.85	0.0000	0.0000	0.0000
0.0075	0.9	0.0000	0.0000	0.0000
0.0075	0.95	0.0000	0.0000	0.0000
0.0075	1	0.0000	0.0000	0.0000

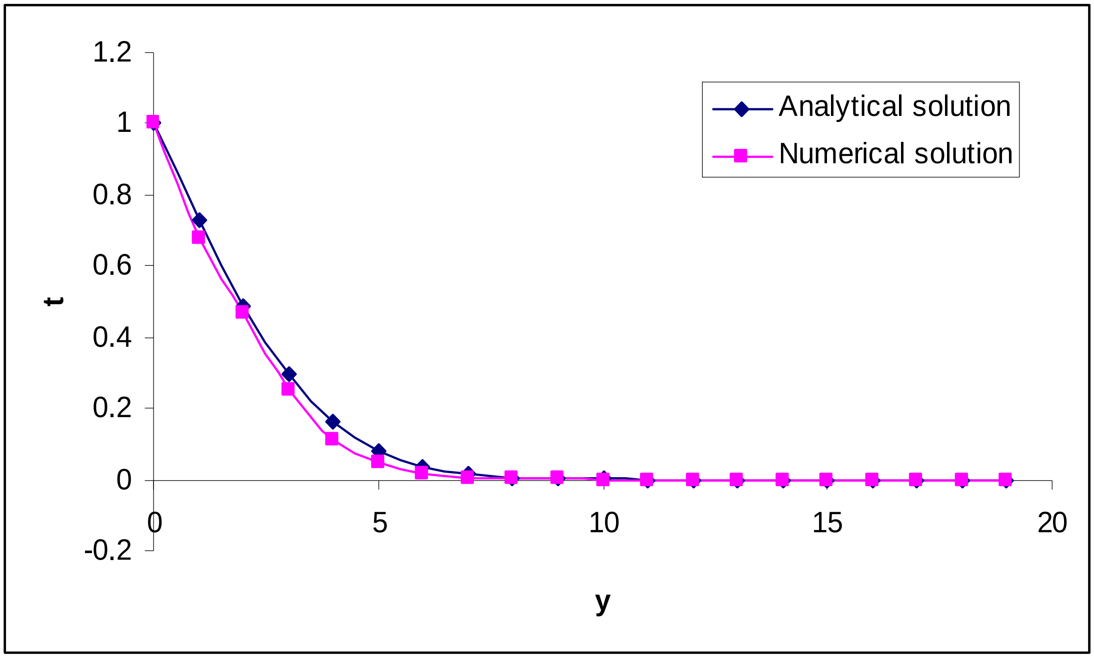

Fig.2. Comparison of Temperature Profiles when P_r=0.7333

Table.2. Comparison of Velocity when P_r=6.75

t	y	Analytical solution	Numerical solution	| Percentage error |
0.005	0	1	1	0.0000
0.005	0.05	0.610230869	0.6300	0.0002
0.005	0.1	0.311560833	0.3816	0.0007
0.005	0.15	0.130586682	0.1644	0.0003
0.005	0.2	0.04433234	0.0586	0.0001
0.005	0.25	0.012075089	0.0186	0.0001
0.005	0.3	0.002621086	0.0055	0.0000
0.005	0.35	0.000451214	0.0015	0.0000
0.005	0.4	6.13822E-05	0.0004	0.0000
0.005	0.45	6.58E-06	0.0001	0.0000
0.005	0.5	5.55E-07	0.0000	0.0000
0.005	0.55	3.68E-08	0.0000	0.0000
0.005	0.6	1.91E-09	0.0000	0.0000
0.005	0.65	7.77E-11	0.0000	0.0000
0.005	0.7	2.48E-12	0.0000	0.0000
0.005	0.75	6.17E-14	0.0000	0.0000
0.005	0.8	1.20E-15	0.0000	0.0000
0.005	0.85	1.83E-17	0.0000	0.0000
0.005	0.9	2.18E-19	0.0000	0.0000
0.005	0.95	2.03E-21	0.0000	0.0000
0.005	1	1.47E-23	0.0000	0.0000

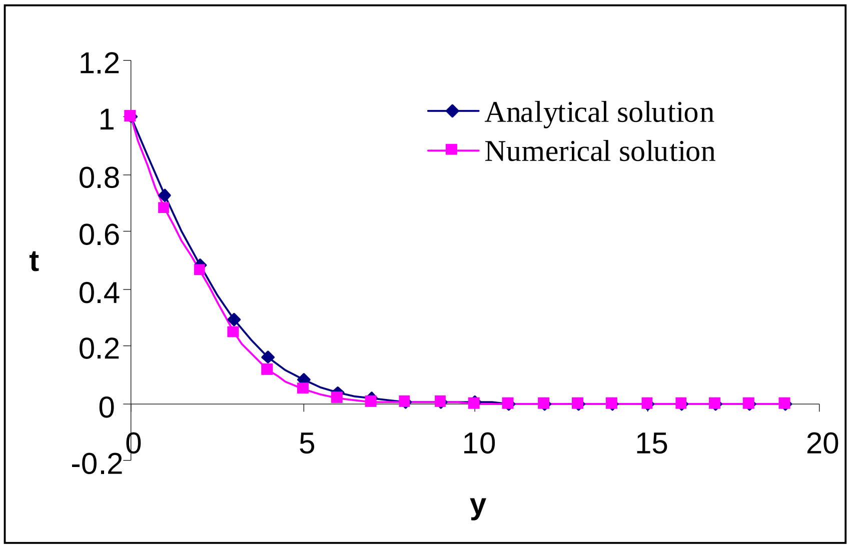

Fig.3. Comparison of Velocity when P_r=6.75

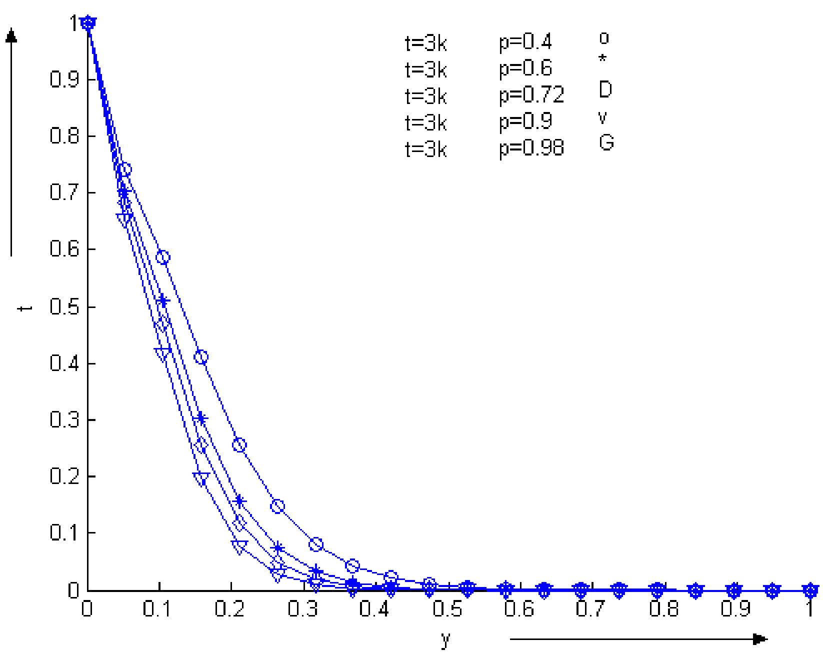

Fig.4. Temperature profiles for fixed‘t’ and P_r<1

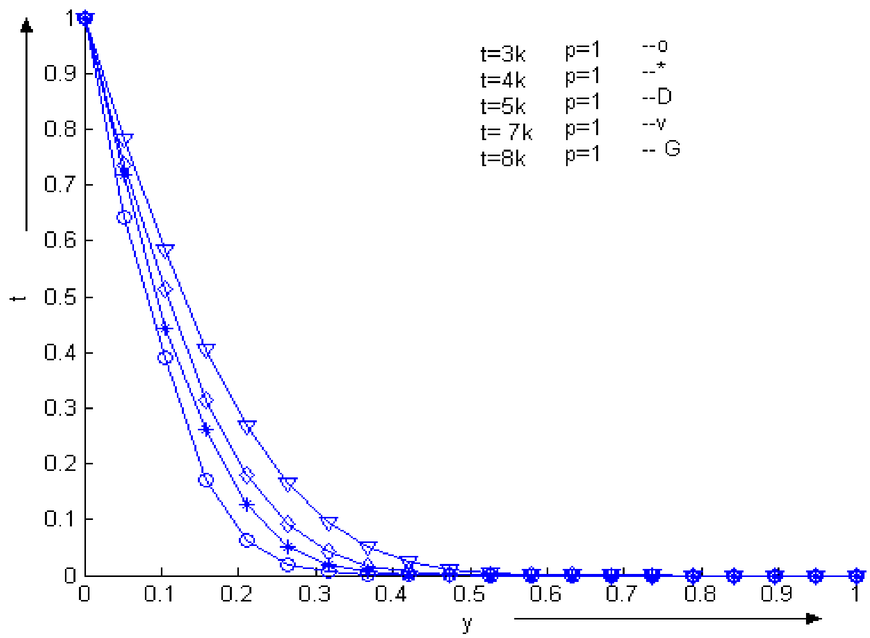

Fig.5. Temperature profiles for different 't' and for fixed P_r=1

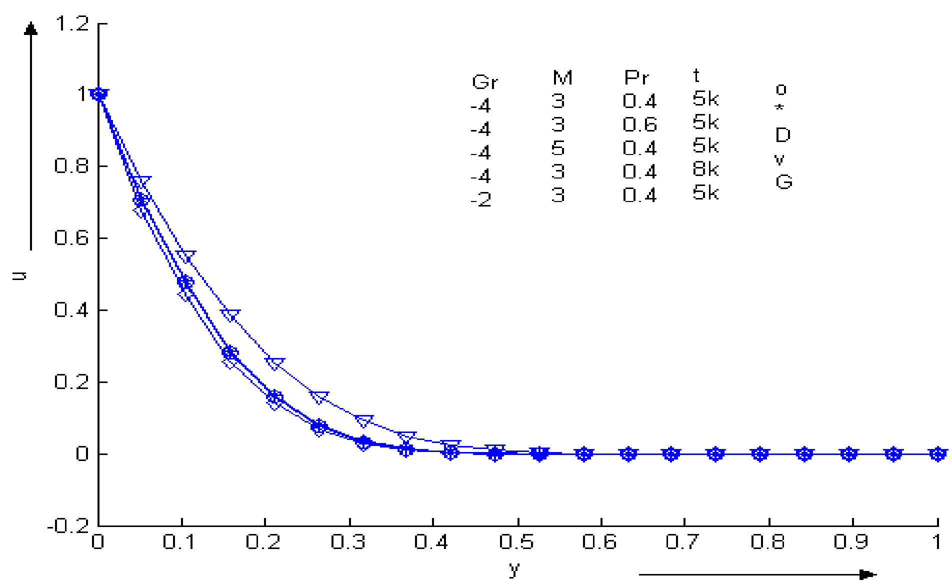

Fig (6) Velocity profiles for P_r<1 and different G_r, t, m (for heating of plate)

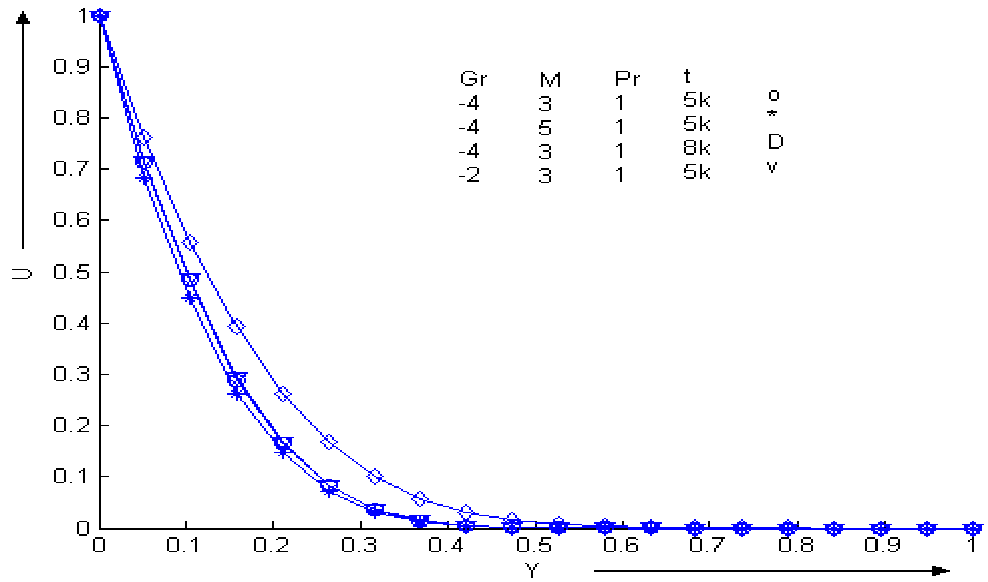

Fig.7. Velocity profiles for P_r=1 and different G_r, t, m (for heating of plate)

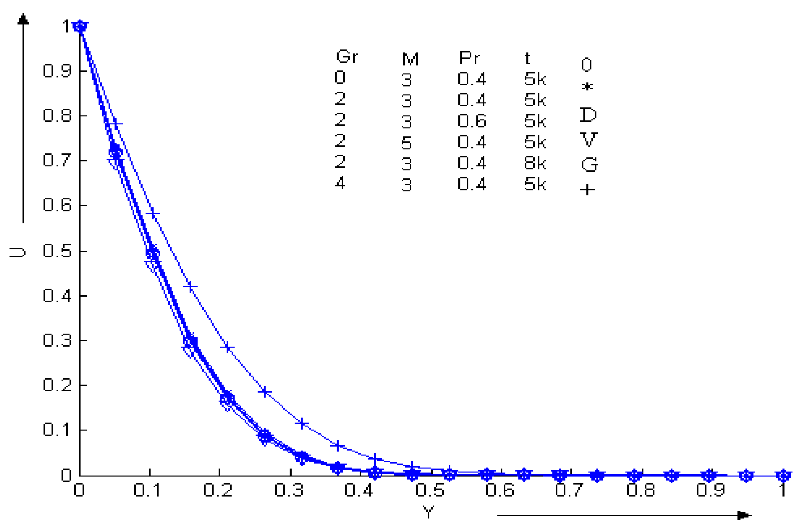

Fig.8. Velocity profiles for P_r<1 and different G_r, M, t (for cooling of the plate)

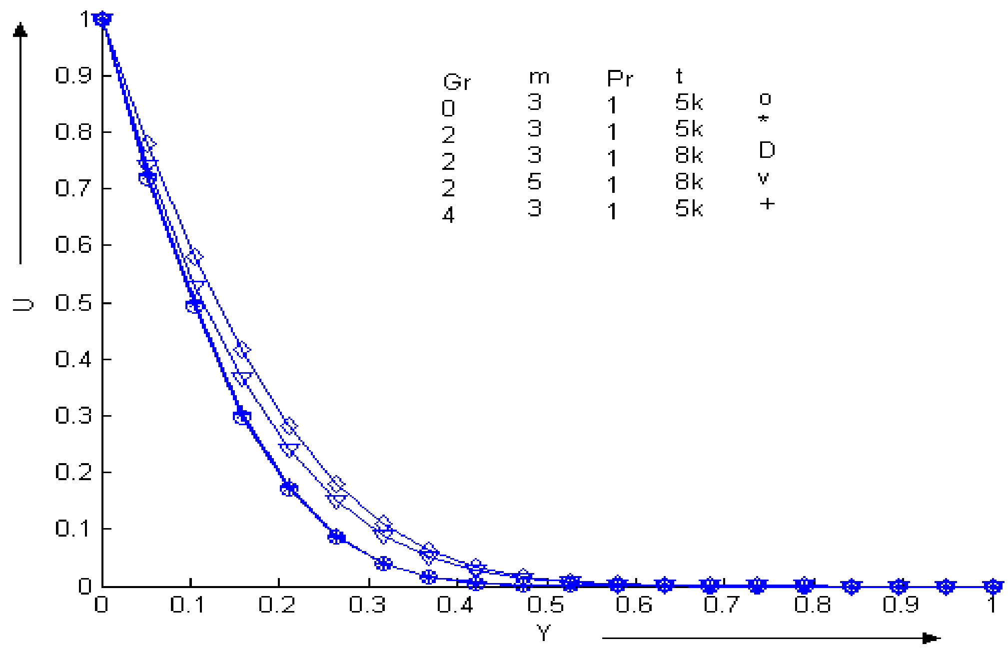

Fig.9. Velocity profiles for P_r=1 and different G_r, t, M (for cooling of the plate)

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