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EFFECTS OF CHEMICAL REACTION ON FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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Abstract

An exact analysis is performed to study the unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of the homogeneous chemical reaction of first-order. The temperature of the plate as well as concentration near the plate is raised linearly with respect to time. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, time and a. It is observed that the velocity increases with increasing values of time or a. But the trend is just reversed with respect to the Scmidt number.

Key words: chemical reaction, accelerated, vertical plate, exponential, heat and mass transfer.
Mathematics Subject Classification : 76R10

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1. Introduction

Diffusion rates can be tremendously altered by chemical reactions. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to concentration. In many chemical engineering processes, there is the chemical reaction between a foreign mass and the fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production.

Chambre and Young [2] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al. [3] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [4]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level. Gupta [5] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [7]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [6]. Basant Kumar Jha et al. [1] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

It is proposed to study the unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. Such a study found useful in chemical process industries such as wire drawing, fibre drawing, food processing and polymer production. The solutions are in terms of exponential and complementary error function.

2. Analysis

First order chemical reaction effects on unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Here the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is also raised with time. It is also assumed that there exists first order chemical reaction between the fluid and the species concentration. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_1 C' \qquad (3)$$

with the following initial and boundary conditions:

$$\begin{array}{llll} & u = 0, & T = T_\infty, & C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0 \\ t' > 0: & u = u_0 \exp(a't'), & T = T_\infty + (T_w - T_\infty)At', & C' = C'_\infty + (C'_w - C'_\infty)At' \quad \text{at} \quad y = 0 \\ & u \to 0, & T \to T_\infty, & C' \to C'_\infty \quad \text{at} \quad y \to \infty \end{array} \qquad (4)$$

where $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - c'_\infty}, \quad Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3}, \tag{5}$$
$$Pr = \frac{\mu C_p}{k}, \quad a = \frac{a'\nu}{u_0^2}, \quad K = \frac{\nu K_1}{u_0^2}, \quad Sc = \frac{\nu}{D}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0 & \text{for all } Y, t \leq 0 & \\ t > 0: & U = \exp(at), & \theta = t, & C = t & \text{at} & Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{at} & Y \to \infty \end{array} \tag{9}$$

3. SOLUTION PROCEDURE

The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\eta^2 Pr)\mathrm{erfc}(n\sqrt{Pr}) - 2\eta\sqrt{\frac{Pr}{\pi}}\exp(-\eta^2 Pr)\right] \tag{10}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt}) \quad - \exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt})\right] \end{aligned} \tag{11}$$

$$
\begin{aligned}
U = {} & \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})\mathrm{erfc}(\eta+\sqrt{at})+\exp(-2\eta\sqrt{at})\mathrm{erfc}(\eta-\sqrt{at})\right]+2e\,\mathrm{erfc}(\eta) \\
& -\frac{dt^2}{6}\Big[(3+12\eta^2+4\eta^4)\mathrm{erfc}(\eta)-\frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2) \\
& \quad -(3+12\eta^2\mathrm{Pr}+4\eta^4\mathrm{Pr}^2)\mathrm{erfc}(\eta\sqrt{\mathrm{Pr}})+\frac{\eta\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}(10+4\eta^2\mathrm{Pr})\exp(-\eta^2\mathrm{Pr})\Big] \\
& -e\exp(ct)\left[\exp(2\eta\sqrt{ct})\mathrm{erfc}(\eta+\sqrt{ct})+\exp(-2\eta\sqrt{ct})\mathrm{erfc}(\eta-\sqrt{ct})\right] \\
& -e\left[\exp(2\eta\sqrt{KtSc})\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{Kt})+\exp(-2\eta\sqrt{KtSc})\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\
& +e\exp(ct)\Big[\exp(2\eta\sqrt{Sc(K+c)t})\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{(K+c)t}) \\
& \quad +\exp(-2\eta\sqrt{Sc(K+c)t})\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{(K+c)t})\Big] \qquad (12) \\
& +2cet\left[(1+2\eta^2)erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\
& -et\left[\exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}+\sqrt{Kt})+\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\
& +\frac{\eta e\sqrt{Sct}}{2\sqrt{K}}\left[\exp(-2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \quad -\exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}+\sqrt{Kt})\right]
\end{aligned}
$$

where, $c=\frac{KSc}{1-Sc}$, $d=\frac{Gr}{1-Pr}$, $e=\frac{Gc}{2c^2(1-Sc)}$ and $\eta=\frac{Y}{2\sqrt{t}}$.

4. Results and Dicussion

In order to get the physical insight into the problem, the numerical computations are carried out for different physical parameters a, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of Prandtl number Pr are chosen such that they represent air (Pr = 0.71). Also, the value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor present in the air. The numerical values of the velocity are computed for different physical parameters like a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of the velocity field for different time (t = 0.2, 0.4, 0.6, 0.8), K = 15, a = 0.5, Gr 2, Gc = 2 are shown in Figure 1. In this case, the velocity increases gradually with respect to time t. Figure 2 illustrates the effect of velocity for different values of the chemical reaction parameter (K = 4,5,10), a = 0.5, Gr = 2= Gc and t = 0.2. The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the relative variation of the velocity with the magnitude of the chemical reaction parameter. The velocity profiles for different (a = 0, 0.2, 0.5, 0.8), K = 15, Gr = Gc = 2 at t = 0.4 are studied and presented in Figure 3. It is observed that the velocity increases with increasing values of a.

Figure 4. represents the effect of velocity at time t = 0.4 for different Schmidt number (Sc = 0.2, 0.3, 0.6, 0.16), Gr=Gc=2 and K=15. The profiles have the common feature that the velocity decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the velocity increases with decreasing values of the Schmidt number.

5. Conclusion

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature and mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, a and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc, a and t. But the trend is just reversed with respect to the Schmidt number.

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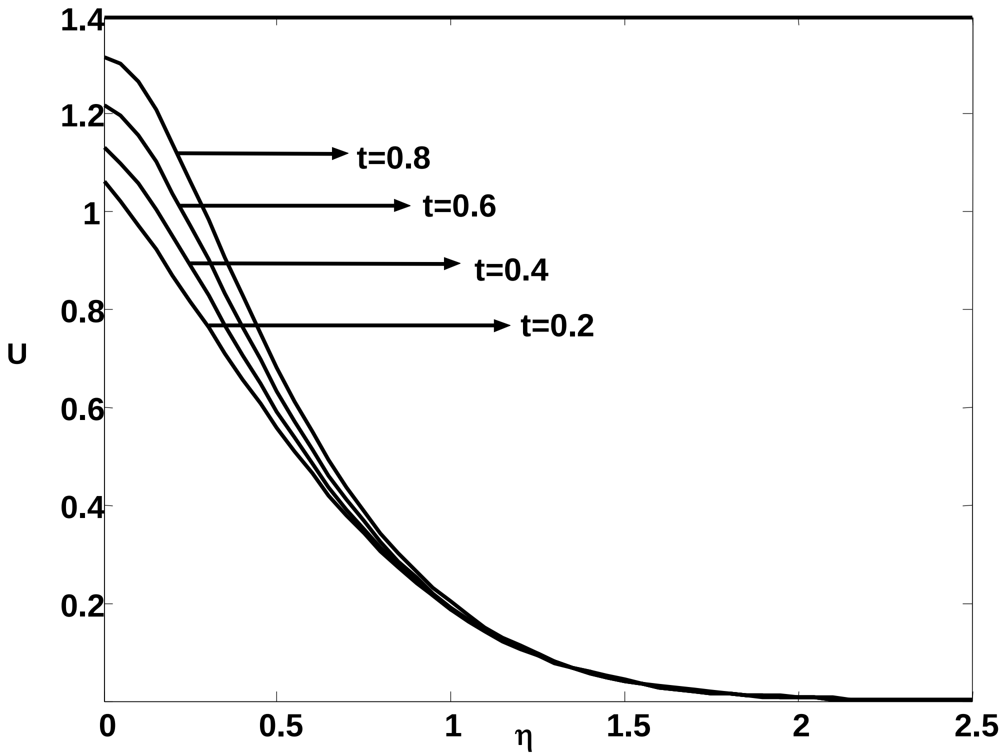

Fig. 1 Velocity profiles for different values of t

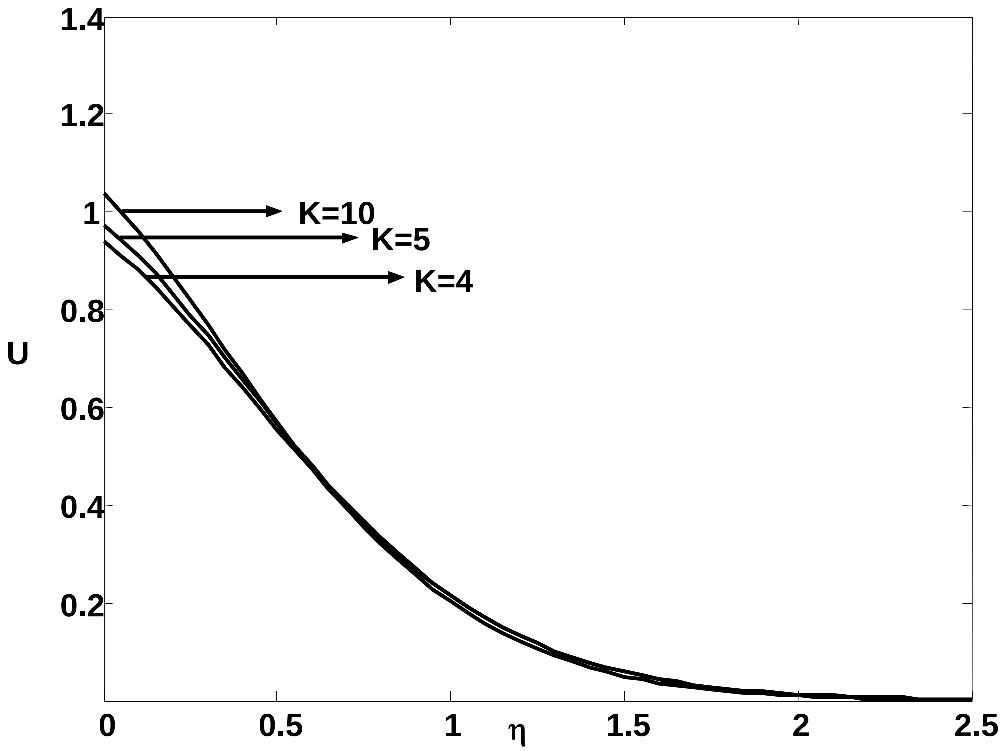

Fig. 2 Velocity profiles for different values of K

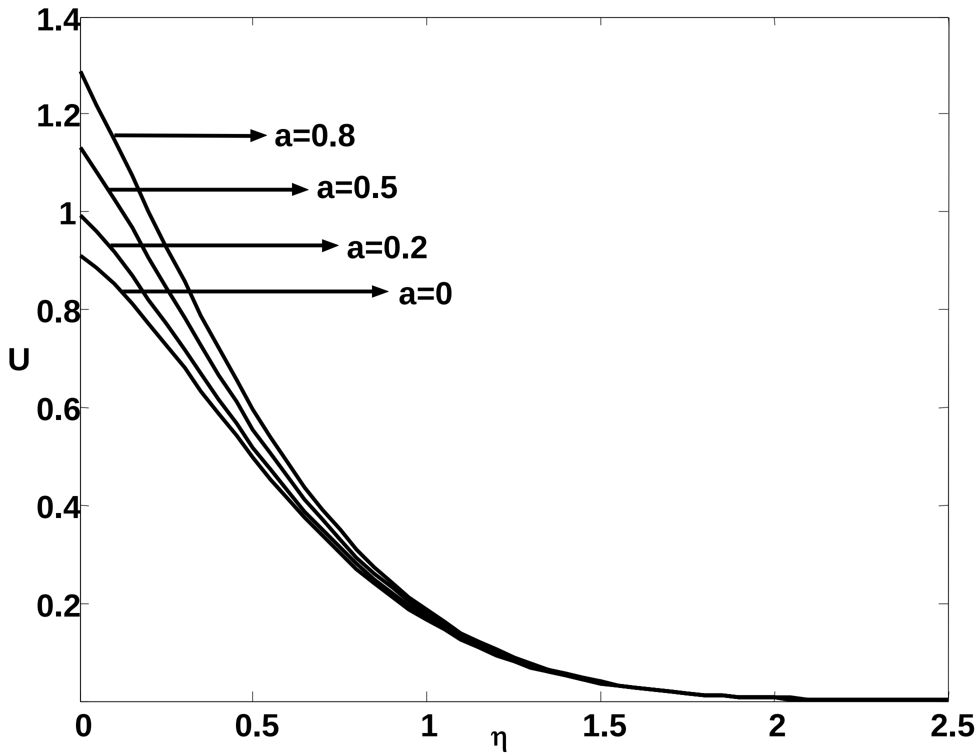

Fig. 3 Velocity profiles for different values of a

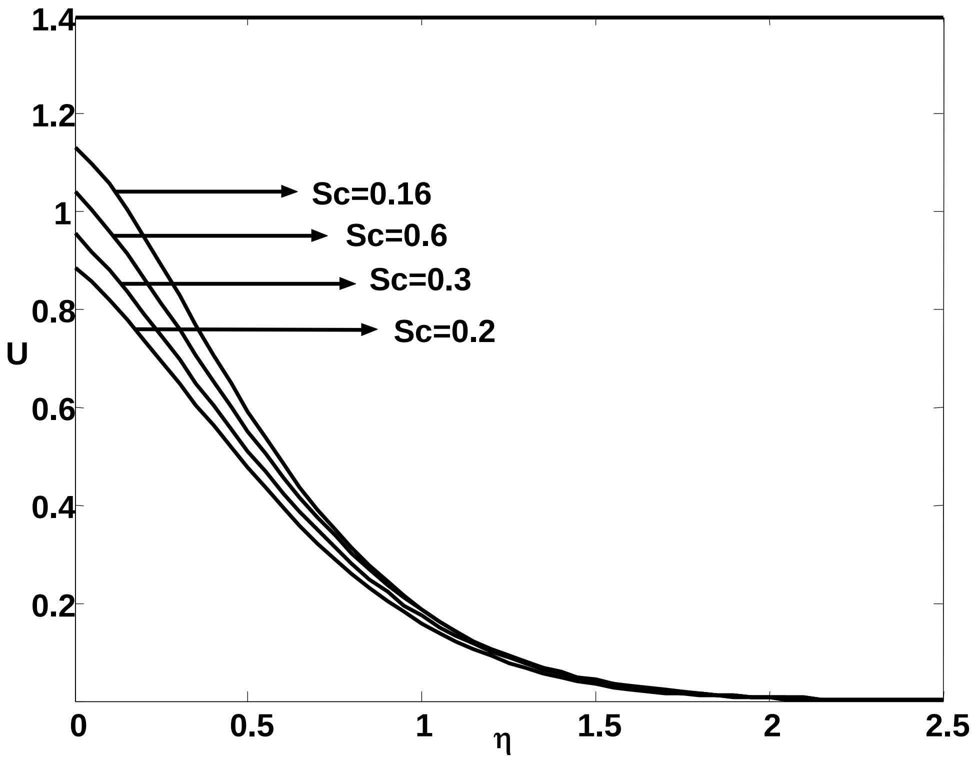

Fig. 4 Velocity profiles for different values of Sc