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# Non-Darcy Convective Heat Transfer In A Vertical Using Finite Element Techniques

**Arjumand Sattar[1], Ahmed Waheedullah[2]**

[1] Department of Mathematics, Deccan College of Engineering & Technology, Darussalam, Hyderabad – 500001, A.P., India. Email: arjumandsattar@yahoo.com

[2] Department. of Mathematics, Nawab Shah Alam Khan College of Engineering & Technology,Hyderabad, A.P., India.



## Abstract

The non-darcy convective heat transfer of a viscous dissipative fluid through a porous medium in a vertical channel with heat generating sources is considered. By using finite element techniques, the expression for velocity distribution, temperature distribution and rate of heat transfer are obtained. Influence of parameters on velocity distribution and temperature distribution are shown graphically. Influence of parameters on rate of heat transfer are presented through tables.

**Key words :** Non-Darcy flow, Vertical channel, Heat sources, Radiation, Eckert number.

## Introduction

Any substance with a temperature above zero transfers heat in the form of radiation. Thermal radiation always exits and can strongly interact with convection in many situations of engineering interest. This topic have been discussed by Arpaci, Selametand and Kao. [2], the influence of radiation on natural or mixed convection is generally stronger than that on forced convection because of the inherent coupling between temperature and flow field. Chawla and Chan [6] studied the effect of radiation heat transfer on thermally developing Poiseuille flow. Natural convection-radiation interaction is studied by Lauriat Prasad [8] for a vertical porous cavity.

Akiyama and Chang [1] numerically analysed influence of gray surface radiation on convection of non-participating fluid in a rectangular enclosed space. Bevkermann, Viskanta and Ramadhyani [3] numerically studied non-darcy convection in a vertical porous medium. Chamkha et. al. [4] analyze the effect of radiation heat transfer on flow and thermal fields in the presence of a magnetic field for horizontal and inclined plates. Shohel Mahmud [10] studied the effects of radiation heat transfer on magnetohydrodynamic mixed convection through a vertical channel packed with fluid saturated porous substances. Rukhsana, Narasimha and Prasada [9] analyze the non-darcy convective heat transfer in a vertical channel with constant heat flux.

In this paper, we investigate the effect of radiation on the Non-Darcy convective heat transfer flow of a viscous dissipative fluid through a porous medium confined in a vertical channel in the presence of heat generating sources. The non-linear coupled equations governing the flow and heat transfer have been solved by finite element technique. The velocity distribution, temperature distribution and rate of heat transfer are evaluated for different set of parameters.

## Formulation of the Problem:

We consider the flow of a viscous, incompressible fluid through a porous medium confined in a vertical channel in the presence of heat sources. A Cartesian coordinate system O(x,y,z) is used so that the boundaries are taken at $y = \pm L$, where 2L is the distance between the walls. Boussinesq approximation is used so that the density variation is taken only in the buoyancy force term. The plate sat $y = -L$ is maintained at constant temperature T1 and the plate at $y = +L$ is maintained at a constant flux. The temperature gradient in the flow is sufficient to cause natural convection in the flow field. A constant axial pressure gradient is also imposed so that this resultant flow is a mixed convection flow. We also dissipation into account in the energy equation. The porous matrix is assumed to be isotropic and homogeneous with constant porosity and the effective thermal diffusivity. The equations governing the flow and heat transfer are

Momentum equation:

$$-\frac{\partial p}{\partial x}+\left(\frac{\mu}{\delta}\right)\frac{\partial^2 u}{\partial y^2}-\left(\frac{\mu}{k}\right)u-\left(\frac{\rho\delta F}{\sqrt{k}}\right)u^2-\rho g=0 \tag{1}$$

Energy equation :

$$\left(\rho_0 C_p u\right)\frac{\partial T}{\partial x}=\lambda\frac{\partial^2 T}{\partial y^2}+Q_1-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2}$$

Equation of state :

$$\rho-\rho_e=-\beta\left(T-T_e\right) \tag{3}$$

Boundary conditions :

$u = 0$ on $y = \pm L$

T = T1 on y = -L

$$\frac{\partial T}{\partial y} = -q \quad \text{on} \quad y = +L \tag{4}$$

Where T temperature of the fluid, ρ is the density of the fluid, $C_p$ is the specific heat at constant pressure, k is the permeability of the porous medium, μ is the coefficient of viscosity of the fluid, δ is the porosity of the medium, β is the coefficient of thermal expansion, λ is the coefficient of thermal conductivity, F is the function that depends on the Reynolds number and microstructure of porous medium and $Q_1$ is the strength of the heat source.

The axial gradient $\frac{\partial T}{\partial x}$ is assumed to be a constant, say A.

Invoking Rosseland approximation for radiation flux, we get

$$q_r = -\left(\frac{4\sigma^*}{\beta_R}\right)\frac{\partial (T')^4}{\partial y}$$

and linearising $(T')^4$ about $T_e$ by using taylor's expansion and neglecting higher order terms we get

$$(T')^4 \approx 4T_e^4 T - 3T_e^4$$

where $\sigma^*$ is the Stefan-Boltzman constant and $\beta_R$ is the mean absorbing coefficient.
Introducing Non-dimensional quantities

$$(x', y') = \frac{(x, y)}{L}, \quad u' = \frac{uL}{\nu}, \quad p' = \frac{p\delta L^2}{\rho \nu^2}, \quad t = \frac{T - T_2}{T_1 - T_2} \tag{5}$$

Substituting (5) in (1), (2), (3) & (4), we get

$$-\frac{dp}{dx} = \frac{d^2u}{dy^2} - \delta D^{-1}u - \delta Gt \tag{6}$$

$$PN_t u = \frac{d^2t}{dy^2} + \alpha + PEc\left(\frac{du}{dy}\right)^2 + \frac{4}{3n}\left(\frac{d^2t}{dy^2}\right) \tag{7}$$

Boundary conditions :

$$u(\pm 1) = 0, \quad t(-1) = 1, \quad \frac{\partial t}{\partial y}(+1) = -1 \tag{8}$$

## Solution of the Problem

Assume the parameter $\delta$ to be small

$$u(y) = u_0(y) + \delta u_1(y) + \delta^2 u_2(y) + \cdots\cdots \tag{9}$$

$$t(y) = t_0(y) + \delta t_1(y) + \delta^2 t_2(y) + \cdots\cdots \tag{10}$$

Substituting (9) & (10) in (6), (7) & (8) and equating the like powers of $\delta$, we get,
Zero order :

$$\frac{d^2u_0}{dy^2} = \pi \tag{11}$$

$$\frac{d^2t_0}{dy^2} + \alpha = PN_t u_0 + PEc\left(\frac{du_0}{dy}\right)^2 \tag{12}$$

First order :

$$\frac{d^2u_1}{dy^2} - \beta^2 u_1 = -Gt_0 \tag{13}$$

$$\frac{d^2t_1}{dy^2} = PN_t u_1 + 2PEc\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right) \tag{14}$$

Second order :

$$\frac{d^2u_2}{dy^2} - \beta^2 u_2 = n(u_0)^2 - Gt_1 \tag{15}$$

$$\frac{d^2t_2}{dy^2} = PN_t u_2 + PEc\left(\left(\frac{du_1}{dy}\right)^2 + 2\left(\frac{du_0}{dy}\right)\left(\frac{du_2}{dy}\right)\right) \tag{16}$$

Boundary conditions :

$$u_0(+1) = 0, \quad u_0(-1) = 1, \quad \frac{dt_0}{dy}(+1) = -1, \quad t_0(-1) = 1 \tag{17}$$

$$u_1(\pm 1)=0, \quad t_1(-1)=0, \quad \frac{dt_1}{dy}(+1)=0 \tag{18}$$

$$u_2(\pm 1)=0, \quad t_2(-1)=0, \quad \frac{dt_2}{dy}(+1)=0 \tag{19}$$

The equations (11) - (16) are ordinary differential equations with boundary conditions (17) - (19). Through finite element technique, using Galerkin method, the solution of $u_0(y)$, $u_1(y)$, $u_2(y)$, $t_0(y)$, $t_1(y)$ & $t_2(y)$ is given by

$$u_0 = \frac{\pi}{2}\left(y^2-1\right) \tag{20}$$

$$u_1 = A_6\left(y^2-1\right) \tag{21}$$

$$u_2 = A_{12}\left(y^2-1\right) \tag{22}$$

$$t_0 = A_3\left(y^3-2y-3\right)-y \tag{23}$$

$$t_1 = A_9\left(y^3-2y-3\right) \tag{24}$$

$$t_2 = A_{16}\left(y^3-2y-3\right) \tag{25}$$

Velocity distribution is given by

$$u(y) = A_{17}\left(y^2-1\right) \tag{26}$$

Temperature distribution is given by

$$t(y) = A_{18}\left(y^2-2y-3\right)-y \tag{27}$$

Rate of heat transfer is given by

$$Nu = \left(\frac{dt}{dy}\right)_{y=-1} = A_{18}\left(2y-2\right)-1 \tag{28}$$

where $A_1$ to $A_{18}$ are constants are not included here for the sake of brevity.
The numerical values of u(y), t(y) and Nu are carried out using C++ language.

## Results and Discussion

For the calculation of velocity distribution, temperature distribution and rate of heat transfer for the non-darcy convective hest transfer of a viscous dissipative fluid through a porous medium in a vertical channel with heat generating sources, using finite element technique, Galerkin method. Influence of different values of parameters on velocity distribution, temperature distribution and rate of heat transfer is studied in the present investigation. Here we considered the absolute values of velocity distribution, temperature distribution and rate of heat transfer.

It is observed from fig : 1 that the velocity u(y) increases with increase in $G > 0$ and increases with decreases in $G < 0$, at $\alpha = 2$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 2 that the velocity u(y) increases with increase in $\alpha$, at $G = 3*103$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 3 that the velocity u(y) increases with increase in P, at $G = 3*103$, $N_t = 0.5$, $\alpha = 2$ & $Ec = 0.5$. It is observed from fig : 4 that the velocity u(y) increases with increase in $N_t$, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 5 that the velocity u(y) increases with increase in Ec, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $N_t = 0.5$.

It is observed from fig : 6 that the temperature t(y) increases with increase in $G > 0$ and increases with decreases in $G < 0$, at $\alpha = 2$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 7 that the temperature t(y) increases with increase in $\alpha$, at $G = 3*103$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 8 that the temperature t(y) increases with increase in P, at $G = 3*103$, $N_t = 0.5$, $\alpha = 2$ & $Ec = 0.5$. It is observed from fig : 9 that the temperature t(y) increases with increase in $N_t$, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $Ec = 0.5$. It is observed from fig : 10 that the temperature t(y) increases with increase in Ec, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $N_t = 0.5$.

It is observed from Table :1 that the rate of heat transfer Nu increases with increase in $G > 0$ and increases with decreases in $G < 0$, at $\alpha = 2$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from Table : 2 that the rate of heat transfer Nu increases with increase in $\alpha$, at $G = 3*103$, $N_t = 0.5$, $P = 0.71$ & $Ec = 0.5$. It is observed from Table :3 that the rate of heat transfer Nu increases with increase in P, at $G = 3*103$, $N_t = 0.5$, $\alpha = 2$ & $Ec = 0.5$. It is observed from Table : 4 that the rate of heat transfer Nu increases with increase in $N_t$, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $Ec = 0.5$. It is observed from Table : 5 that the rate of heat transfer Nu increases with increase in Ec, at $G = 3*103$, $\alpha = 2$, $P = 0.71$ & $N_t = 0.5$.

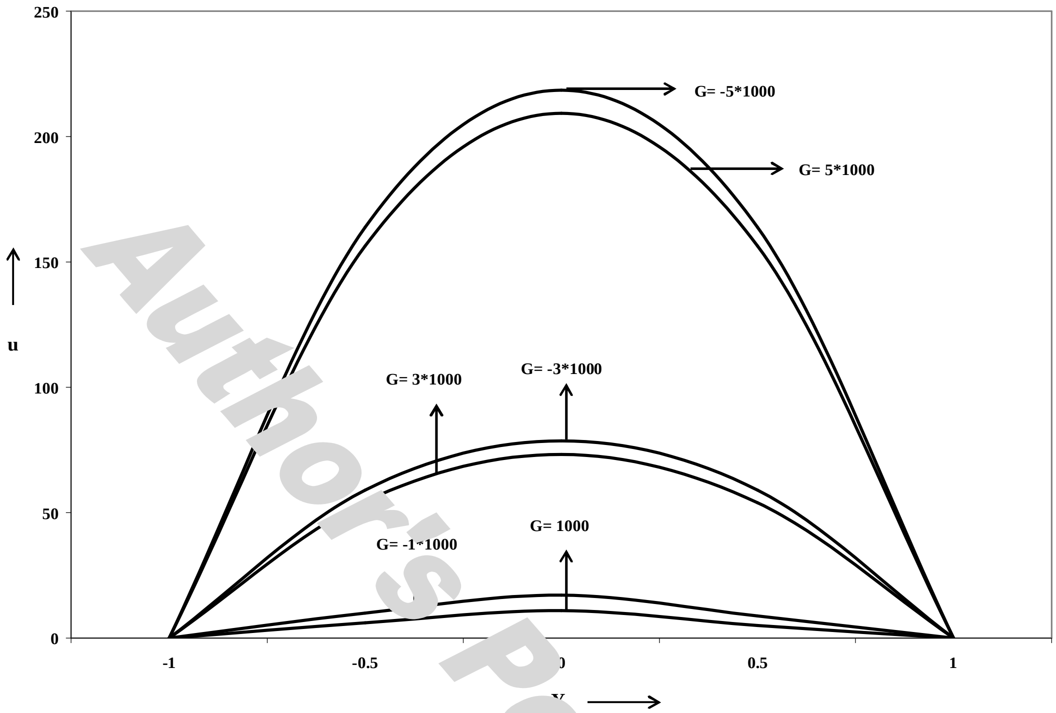


Fig : 1 Variation of u with G

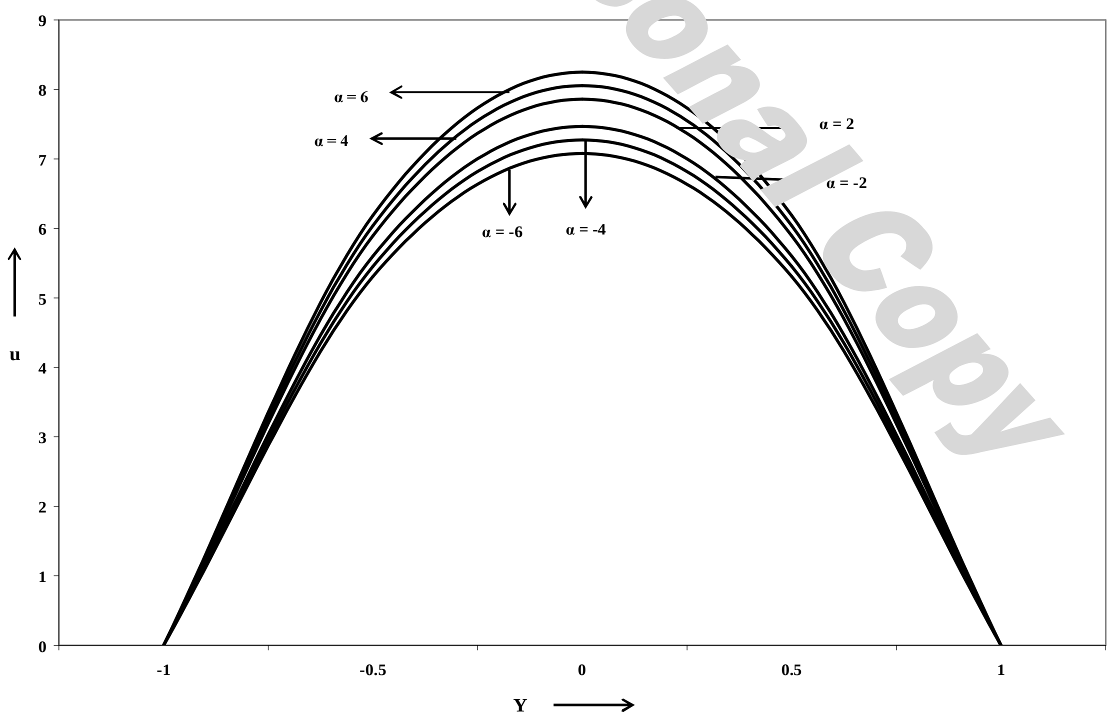


Fig : 2 Variation of u with α

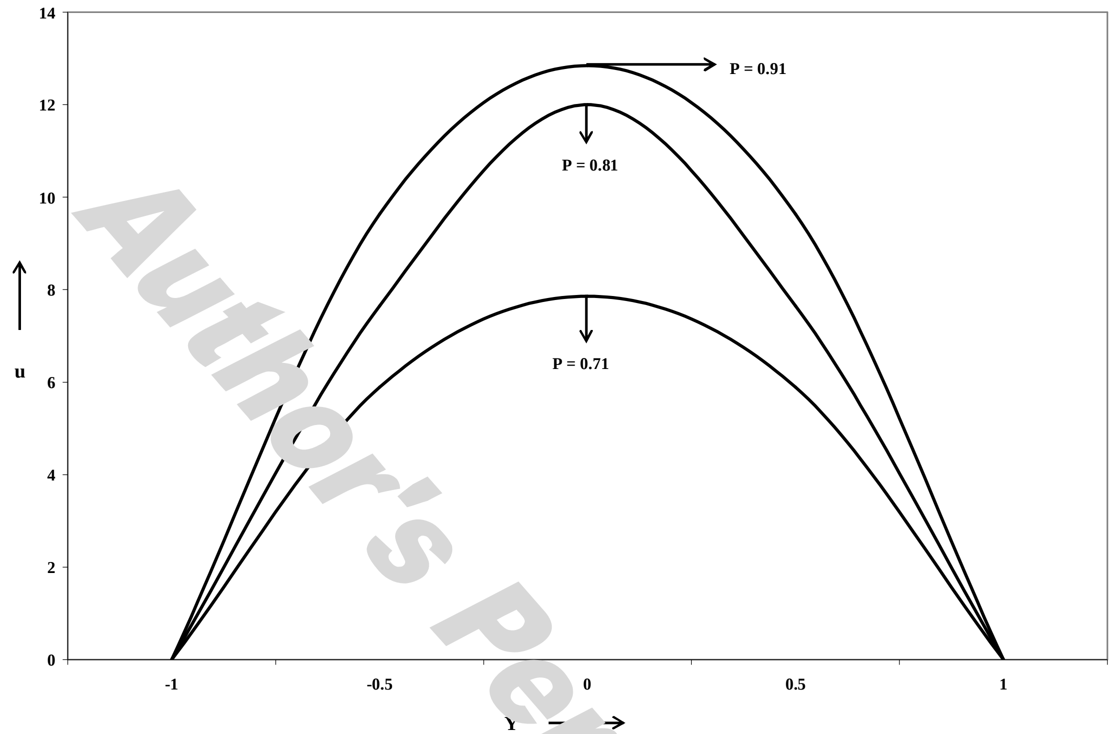


Fig : 3 Variation of u with P

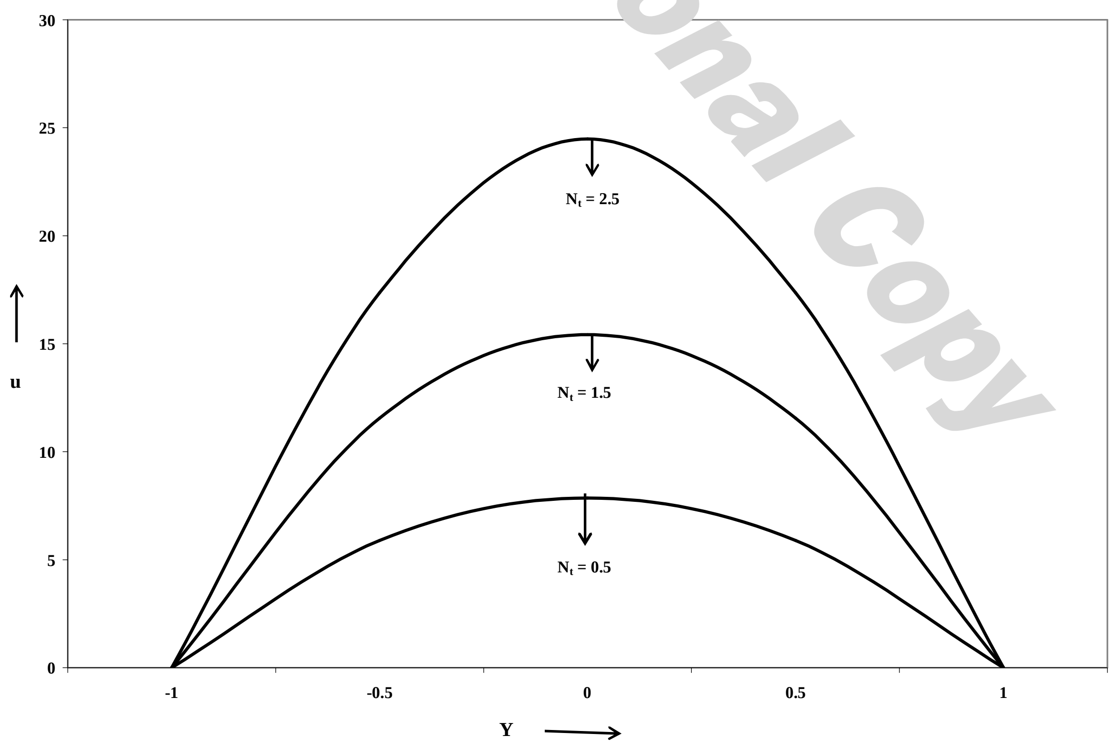


Fig : 4 Variation of u with $N_t$

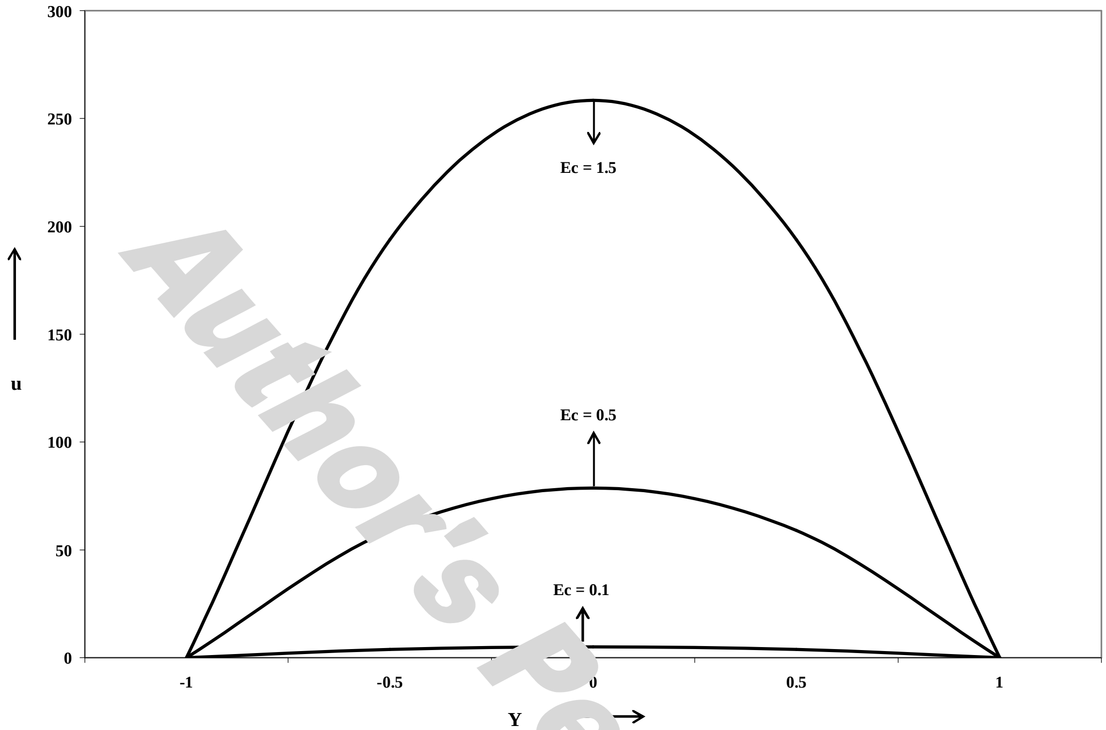


Fig : 5 Variation of u with Ec

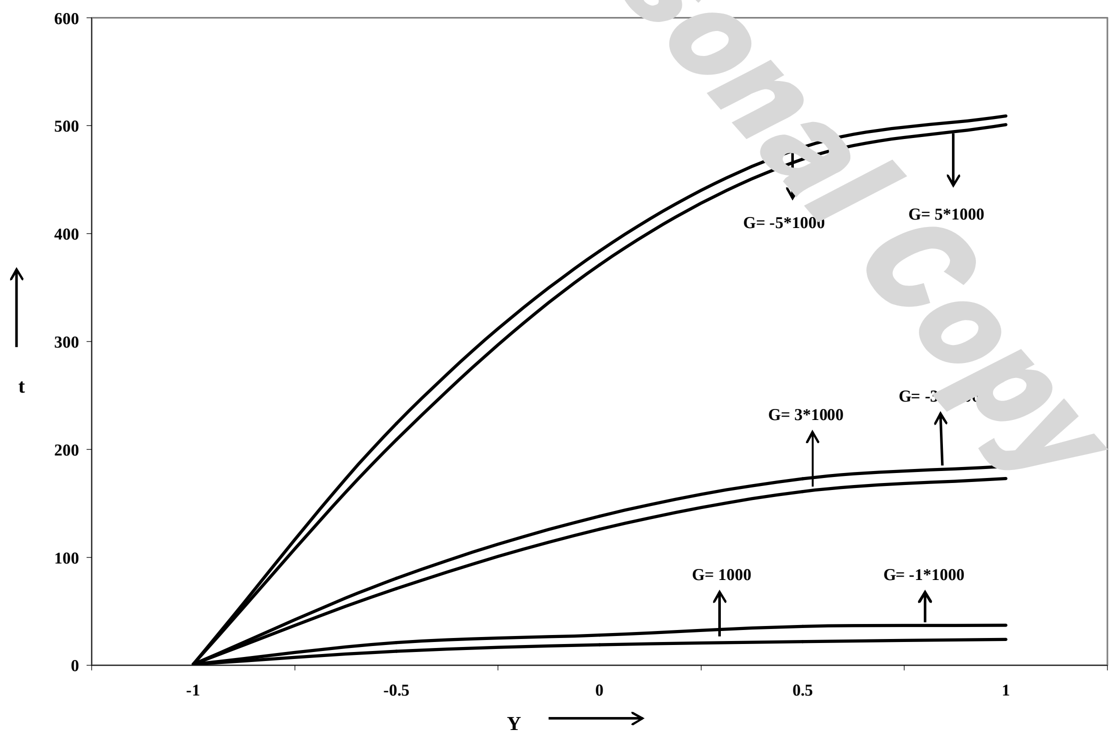


Fig : 6 Variation of t with G

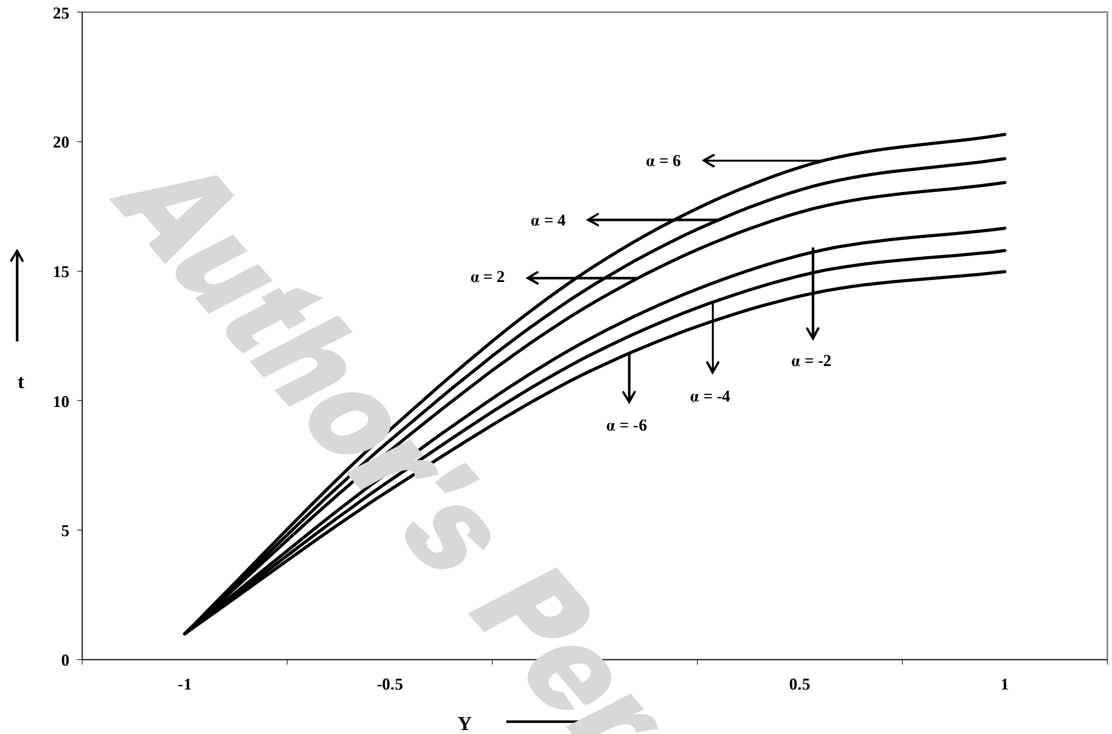


Fig : 7 Variation of t with α

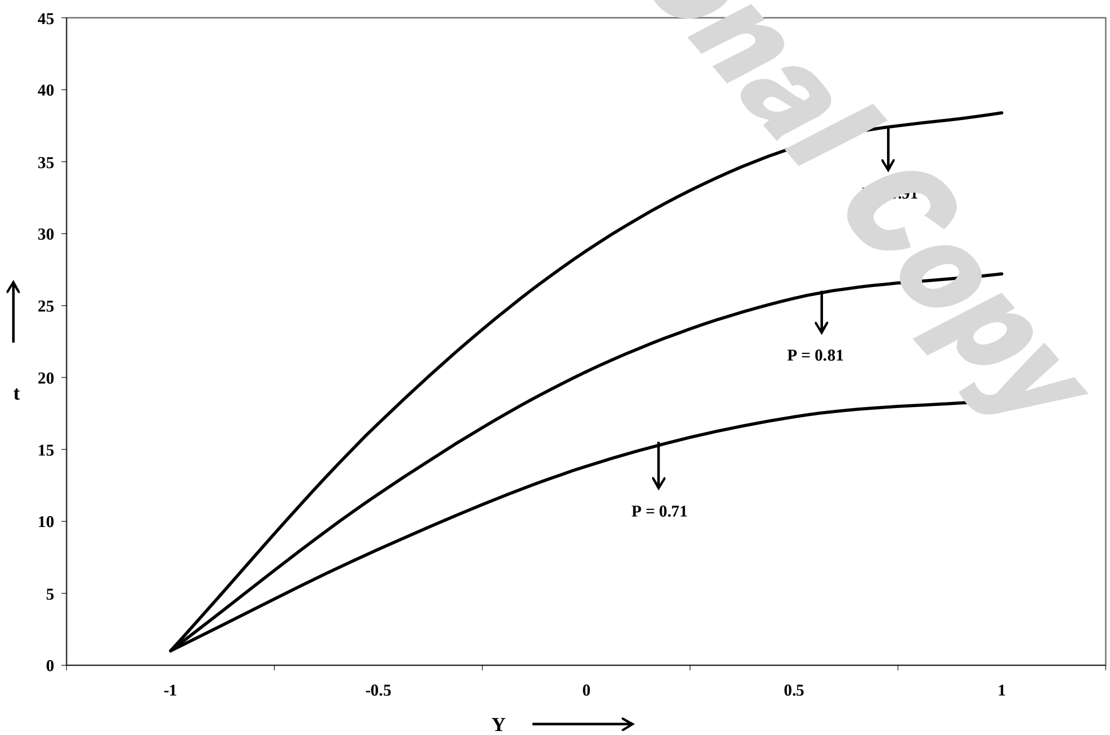


Fig : 8 Variation of t with P

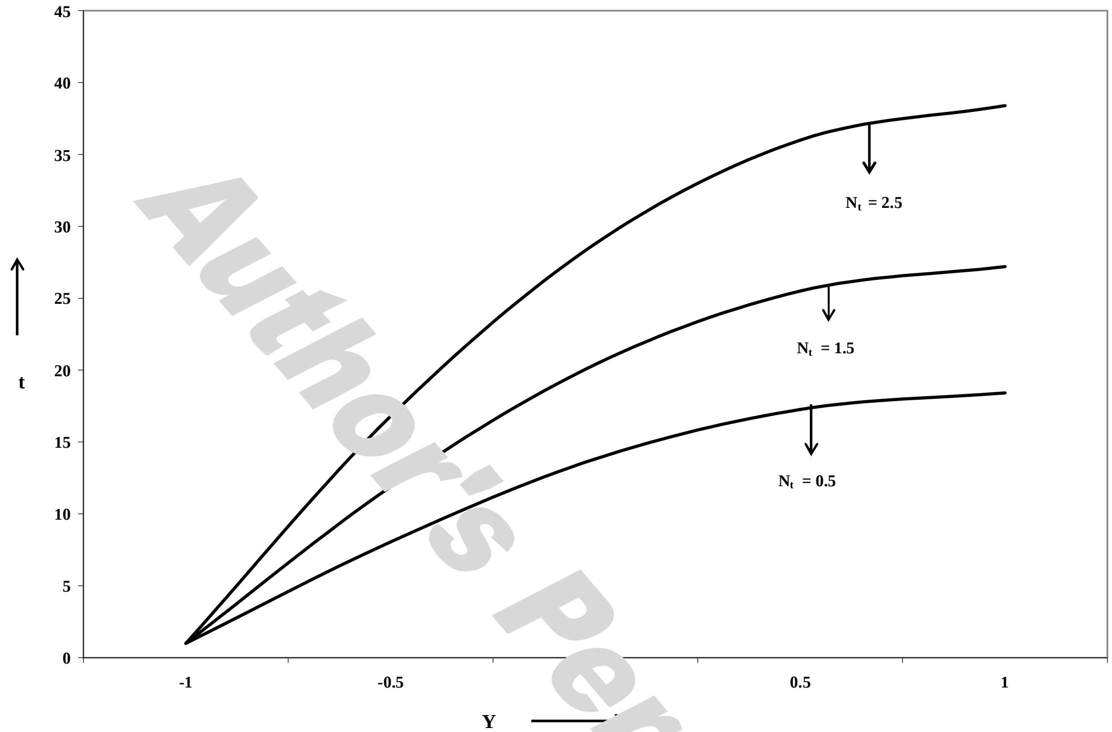


Fig : 9 Variation of t with $N_t$

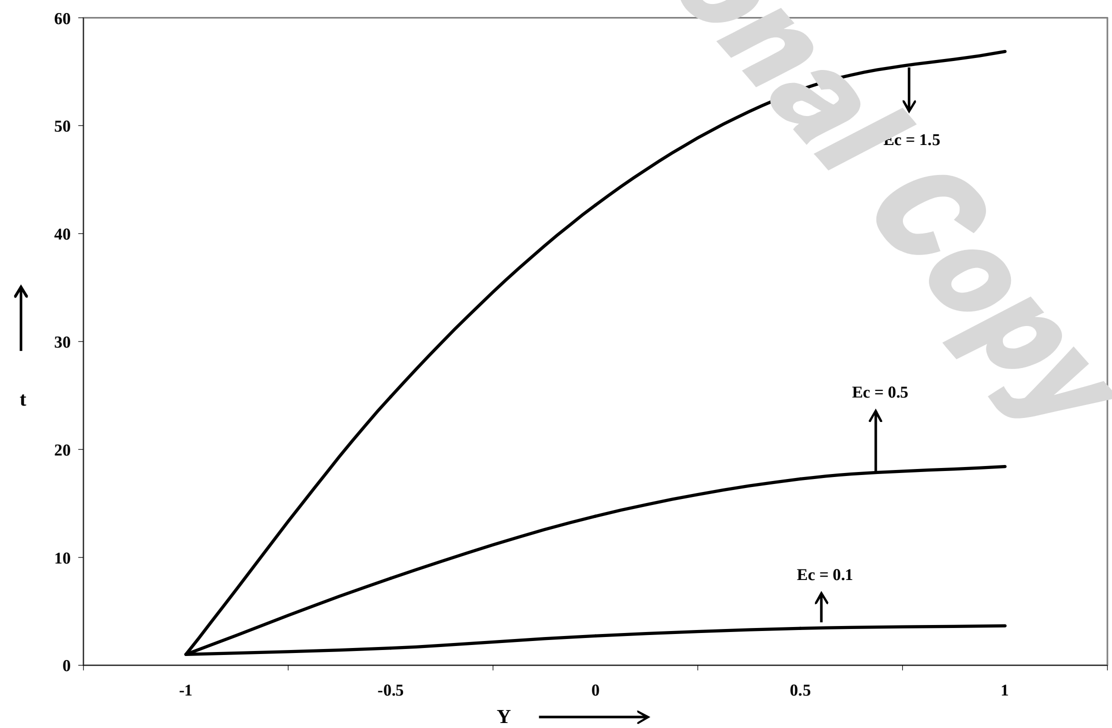


Fig : 10 Variation of t with Ec

| **G** | **Nu ($10^{12}$)** |
|---|---|
| -5000 | 3.199409 |
| -3000 | 1.151789 |
| -1000 | 0.127977 |
| 1000 | 0.127977 |
| 3000 | 1.151789 |
| 5000 | 3.19909 |

Table : 1 Variation of Nu with

| **α** | **Nu ($10^{12}$)** |
|---|---|
| -6 | 0.936294 |
| -4 | 0.988075 |
| -2 | 1.041249 |
| 2 | 1.151779 |
| 4 | 1.209133 |
| 6 | 1.267881 |

Table : 2 Variation of Nu with α

| **P** | **Nu ($10^{12}$)** |
|---|---|
| 0.7 | 1.104529 |
| 0.75 | 1.354248 |
| 0.8 | 1.639022 |
| 0.85 | 1.961155 |
| 0.9 | 2.322948 |

Table 3: Variation of Nu with P

| **$N_t$** | **Nu ($10^{12}$)** |
|---|---|
| 0.5 | 1.151779 |
| 1.0 | 4.469157 |
| 1.5 | 9.95728 |
| 2.0 | 17.62046 |
| 2.5 | 20.74630 |

Table 4: Variation of Nu with $N_t$

| **Ec** | **Nu ($10^{12}$)** |
|---|---|
| 0.1 | 0.228168 |
| 0.15 | 0.342409 |
| 0.35 | 0.802843 |
| 0.5 | 1.151779 |
| 1.5 | 3.554596 |

Table 5: Variation of Nu with Ec

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