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ON THE STABILITY OF A FOUR SPECIES: A PREY-PREDATOR-HOST-COMMENSAL-MUTUAL-SYN ECO-SYSTEM-I (FULLY WASHED OUT STATE)

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Abstract

This paper deals with an investigation on a four Species Syn-Ecological System (Fully Washed out State). The System comprises of a prey (S_1) , a predator (S_2) that survives upon S_1, two hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are mutual. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of one of the sixteen equilibrium points: the fully washed out state is established. The system would be stable if all the characteristic roots are negative, in case they are real, and have negative real parts, in case they are complex. The linearised equations for the perturbations over the equilibrium point are analyzed to establish the criteria for stability and the trajectories illustrated.

Keywords: Commensal, Eco-system, Equilibrium points, Host, Mutual, Prey, Predator, Stability, Trajectories.

1 Introduction

Ecology is a branch of Life and Environment Sciences dealing with the existence of diverse species in the same environment and habitat. It is natural that two or more species living in a common habitat interact in different ways. Mathematical Modeling has been playing an important role for the last half a century in explaining several phenomena concerned with individuals and groups of populations in nature. Significant researches in the area of theoretical ecology had been initiated by Lotka [9] and by Volterra [13]. Since then, several mathematicians and ecologists contributed to the growth of this area of knowledge. The Ecological interactions can be broadly classified as Neutralism, Commensalism, Mutualism, Competition, Ammensalism, Parasitism and Predation.

Broad concepts of Mathematical Modeling can be noted in the books of Meyer [10], Kushing [6]. Some principles in formulating a Mathematical Model and the diverse techniques that could be adopted in mathematical modeling have been illustrated exhaustively by Kapur [5] in his treatise on Mathematical Modeling. Another detailed monograph by Kapur deals exhaustively with diverse topics on Mathematical Modeling in biology and Medical Sciences [4]. The study on competitive eco-systems of two and three species with limited and unlimited resources was done by N.C. Srinivas [12]. Later K. Lakshminarayan [7] studied the two species Prey-Predator ecological models incorporating a partial cover for the Prey and alternate food for the Predator. Recently, Archana Reddy [1] and Bhaskara Rama Sharma [2] investigated on Interacting species and Competitive eco-systems with time delay, employing analytical and numerical techniques. Further study on the stability of a Host – A flourishing commensal species pair with limited resources was done by Phani Kumar, Seshagiri Rao and Pattahi Ramacharyulu [11].

The present investigation is on an analytical study of a four species (S_1, S_2, S_3, S_4) Prey-Predator-Host-Commensal-Mutual-Syn Eco-System. The System comprises of a prey (S_1), a predator (S_2) that survives upon S_1, two hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are mutual. Fig.1 shows a Schematic Sketch of the system under investigation. The stability analysis of a four species eco-system with the interaction between S_3 and S_4 is neutralism was considered by B. Hari Prasad and Pattabhi Ramacharyulu [3]. The model equations of the system in the present case constitute a set of four first order non-linear ordinary differential coupled equations. In all the sixteen equilibrium points of the system are identified and the stability analysis is carried out only for the fully washed out state.
The linearized perturbed equations over the equilibrium states are solved and the trajectories illustrated.

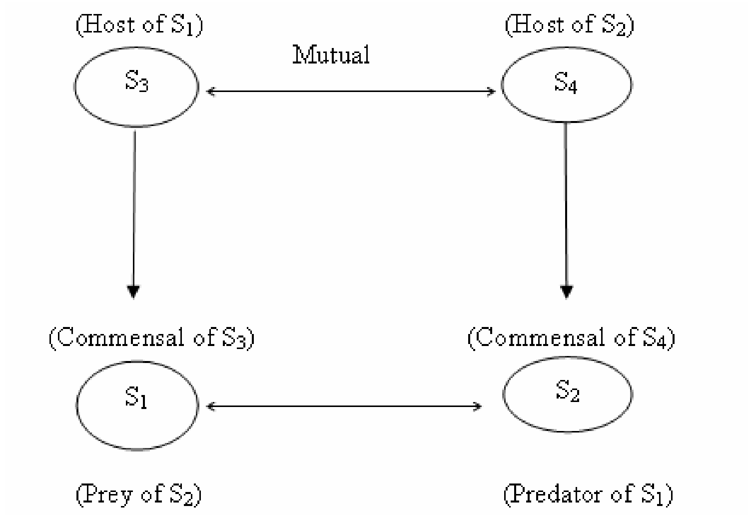

Fig. 1 Schematic Sketch of the Syn Eco-System under investigation

2 Notation Adopted

$N_1(t)$: The population of the prey species (S_1)

$N_2(t)$: The population of the predator species (S_2)

$N_3(t)$: The population of the host species (S_3) of the prey (S_1)

$N_4(t)$: The population of the host (S_4) of the predator (S_2)

t : Time instant

a_1, a_2, a_3, a_4 : Natural growth rates of S_1, S_2, S_3, S_4

$a_{11}, a_{22}, a_{33}, a_{44}$: Self inhibition coefficients of S_1, S_2, S_3, S_4

a_{12}, a_{21} : Interaction (prey-predator) coefficients of S_1 due to S_2 and S_2 due to S_1

a_{13} : Coefficient for commensal for S_1 due to the host S_3

a_{24} : Coefficient for commensal for S_2 due to the host S_4

a_{34} : Coefficient for mutual for S_3 due to S_4

a_{43} : Coefficient for mutual for S_4 due to S_3

$K_i = \frac{a_i}{a_{ii}}$: *Carrying* capacity of S_i, i=1,2,3,4

Further the variables N_1, N_2, N_3, and N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4, a_{11}, a_{22}, a_{33}, a_{44}, a_{12}, a_{21}, a_{13}, a_{24}, a_{34}, a_{43} are assumed to be non-negative constants.

3 Basic Equations

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2 + a_{13} N_1 N_3 \qquad (2.1)$$

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{21} N_1 N_2 + a_{24} N_2 N_4 \qquad (2.2)$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2 + a_{34} N_3 N_4 \qquad (2.3)$$

$$\frac{dN_4}{dt} = a_4 N_4 - a_{44} N_4^2 + a_{43} N_3 N_4 \qquad (2.4)$$

4 Equilibrium states

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0, \qquad i = 1,2,3,4 \qquad (3.1)$$

are given in the following table.

Table 1: Equilibrium states

S.No.	EQUILIBRIUM STATES	EQUILIBRIUM POINT
1	Fully washed out state	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
2	Only the prey S_1 survives	$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
3	Only the predator S_2 survives	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$
4	Only the host (S_3) of S_1 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
5	Only the host (S_4) of S_2 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
6	Prey (S_1) and the predator (S_2) survives	$\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_2} = \frac{a_2 a_{11} + a_1 a_{21}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_3} = 0, \overline{N_4} = 0$

7	Predator (S_2) and the host (S_4) of S_2 washed out	$\overline{N_1}=\frac{a_1a_{33}+a_3a_{13}}{a_{11}a_{33}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
8	Predator (S_2) and the host (S_3) of S_1 washed out	$\overline{N_1}=\frac{a_1}{a_{11}}, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$
9	Prey (S_1) and the host (S_4) of S_2 washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
10	Prey (S_1) and the host(S_3) of S_1 washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2a_{44}+a_4a_{24}}{a_{22}a_{44}}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$
11	Prey (S_1) and the predator (S_2) washed out	$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=\frac{\alpha_2}{\alpha_1}, \overline{N_4}=\frac{\alpha_3}{\alpha_1}$ where $\alpha_1=a_{33}a_{44}-a_{34}a_{43}$ $\alpha_2=a_3a_{44}+a_4a_{34}$ $\alpha_3=a_4a_{33}+a_3a_{43}$
12	Only the host (S_4) of S_2 washed out	$\overline{N_1}=\frac{\beta_2}{\beta_1}, \overline{N_2}=\frac{\beta_3}{\beta_1}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$ *where* $\beta_1=a_{33}(a_{11}a_{22}+a_{12}a_{21})$ $\beta_2=a_1a_{22}a_{33}+a_3a_{13}a_{22}-a_2a_{12}a_{33}$ $\beta_3=a_2a_{11}a_{33}+a_1a_{21}a_{33}+a_3a_{13}a_{21}$
13	Only the host(S_3) of S_1 washed out	$\overline{N_1}=\frac{\theta_2}{\theta_1}, \overline{N_2}=\frac{\theta_3}{\theta_1}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$ *where* $\theta_1=a_{44}(a_{11}a_{22}+a_{12}a_{21})$ $\theta_2=a_1a_{22}a_{44}-a_2a_{12}a_{44}-a_4a_{12}a_{24}$ $\theta_3=a_2a_{11}a_{44}+a_4a_{11}a_{24}+a_1a_{21}a_{44}$
14	Only the Predator (S_2)washed out	$\overline{N_1}=\frac{\psi}{a_{11}\alpha_1}, \overline{N_2}=0, \overline{N_3}=\frac{\alpha_2}{\alpha_1}, \overline{N_4}=\frac{\alpha_3}{\alpha_1}$ *where* $\psi=a_1\alpha_1+a_{13}\alpha_3$
15	Only the prey (S_1) washed out	$\overline{N_1}=0, \overline{N_2}=\frac{\delta}{a_{22}\alpha_1}, \overline{N_3}=\frac{\alpha_2}{\alpha_1}, \overline{N_4}=\frac{\alpha_3}{\alpha_1}$ where $\delta=a_2\alpha_1+a_3a_{24}a_{43}+a_4a_{24}a_{33}$
16	The co-existent state (or) Normal steady state	$\overline{N_1}=\frac{\sigma_2}{\sigma_1}, \overline{N_2}=\frac{\sigma_3}{\sigma_1}, \overline{N_3}=\frac{\alpha_2}{\alpha_1}, \overline{N_4}=\frac{\alpha_3}{\alpha_1}$ *where* $\sigma_1=(a_{11}a_{22}+a_{12}a_{21})\alpha_1$ $\sigma_2=(-a_1a_{22}+a_2a_{12})\alpha_1+a_3(a_{12}a_{24}a_{43}-a_{13}a_{22}a_{44})$ $+a_4(a_{12}a_{24}a_{33}-a_{13}a_{22}a_{34})$ $\sigma_3=(a_1a_{21}+a_2a_{11})\alpha_1+a_3(a_{11}a_{24}a_{43}+a_{13}a_{21}a_{44})$ $+a_4(a_{11}a_{24}a_{33}+a_{13}a_{21}a_{34})$

The present paper deals with the stability of fully washed out state (1) of the above table only. The stability of the other Equilibrium states will be presented in the forthcoming communications.

5 Stability of the fully washed out equilibrium state (Sl.No. 1 in the above table)

To discuss the stability of equilibrium point $\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$

Let us consider small deviations $u_1(t),\ u_2(t),\ u_3(t),\ u_4(t)$ *from the steady state* i.e.,

$$N_i(t)=\bar{N}_i+u_i(t)\ , \qquad i=1,2,3,4 \tag{4.1}$$

Where $u_i(t)$ is a small perturbation in the species S_i.

Substituting (4.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher *powers of* $u_1,\ u_2,\ u_3,\ u_4$ we get

$$\frac{du_i}{dt}=a_iu_i\ , \qquad i=1,2,3,4 \tag{4.2}$$

The characteristic equation of which is

$$(\lambda-a_1)(\lambda-a_2)(\lambda-a_3)(\lambda-a_4)=0 \tag{4.3}$$

whose roots $a_1,\ a_2,\ a_3,\ a_4$ *are all positive .*

Hence the Fully Washed-out State is *Unstable.*

The solutions of the equations (4.2) are

$$u_i=u_{i0}e^{a_it}, \qquad i=1,2,3,4 \tag{4.4}$$

where u_{10} , u_{20} , u_{30} *and* u_{40} *are the initial values of* $u_1,\ u_2,\ u_3,\ u_4$ *respectively*.

There would arise in all 576 cases depending upon the ordering of the magnitudes of *the growth rates* $a_1,\ a_2,\ a_3,\ a_4$ *and the initial values of the perturbations* $u_{10}(t),\ u_{20}(t),\ u_{30}(t),\ u_{40}(t)$ *of the* species S_1, S_2, S_3, and S_4. Of these 576 situations some typical variations are illustrated in figures 6.1 to 6.10 through respective solution curves that would facilitate to make some reasonable observations and the conclusions are presented here.

6 Perturbation Graphs

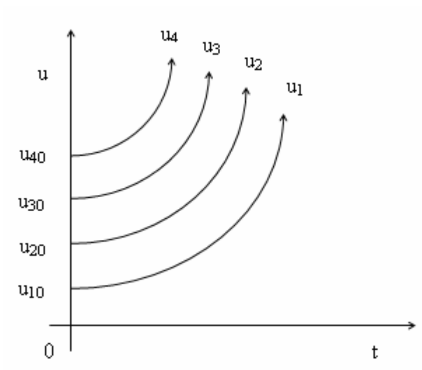

Fig. 6.1

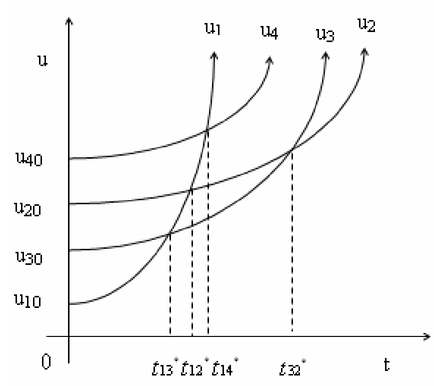

Fig. 6.2

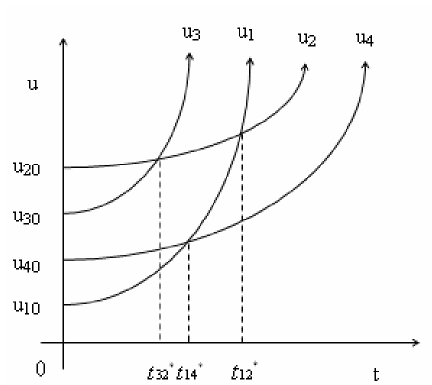

Fig. 6.3

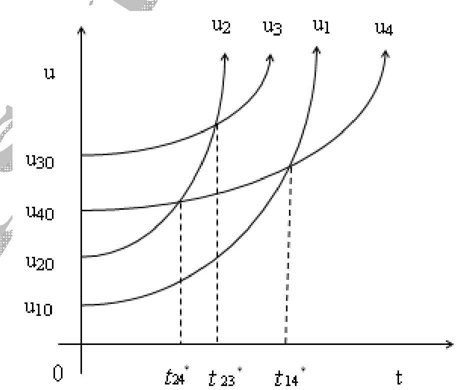

Fig. 6.4

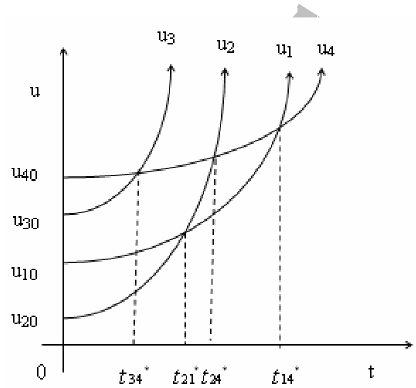

Fig. 6.5

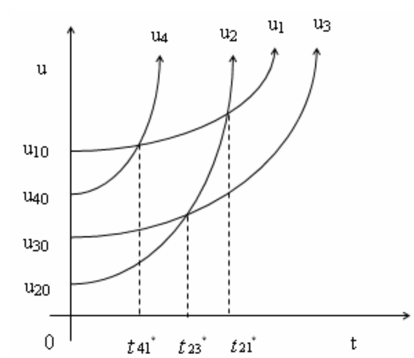

Fig. 6.6

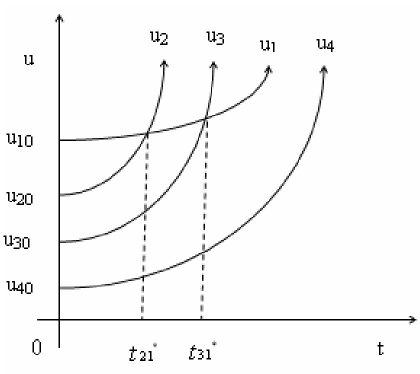

Fig. 6.7

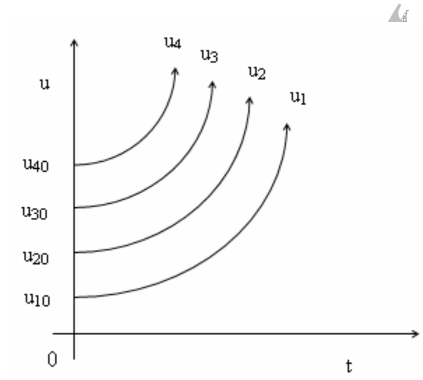

Fig. 6.8

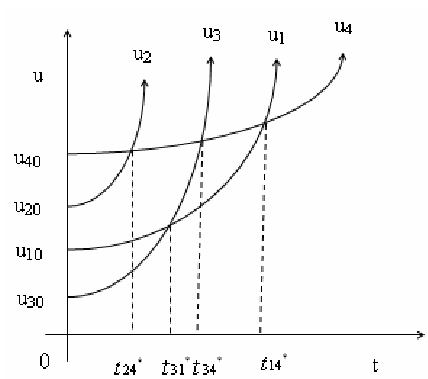

Fig. 6.9

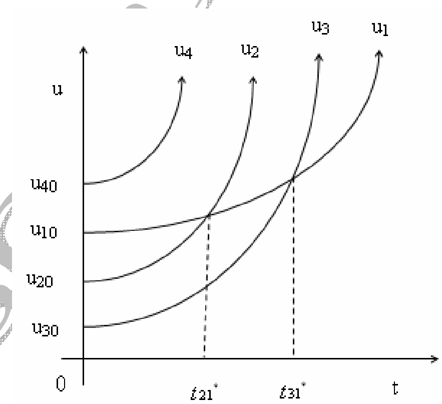

Fig. 6.10

7 Conclusions of the Above Perturbation Graphs

Case (i): *If* $u_{10} < u_{20} < u_{30} < u_{40}, \quad a_1 < a_2 < a_3 < a_4$

In this case prey (S_1) has the least natural growth rate and also the least initial population strength. The host (S_4) of S_2 dominates the prey (S_1), predator (S_2), and host (S_3) of S_1 in natural growth rate as well as in its population strength as shown in Fig. 6.1.

Case (ii): *If* $u_{10} < u_{30} < u_{20} < u_{40}, \quad a_1 < a_4 < a_3 < a_2$

In this case the prey (S_1) has the least natural growth rate as well as the least initial population strength. The host (S_3) of the prey (S_1) initially dominates over the Prey (S_1) till the time instant $t_{13}^* = \frac{1}{a_1 - a_3}\log(\frac{u_{30}}{u_{10}})$ and thereafter the dominance is reversed. Also the predator (S_2) dominates over the prey (S_1) and the host (S_3) of the prey (S_1) till the time instant $t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$, $t_{32}^* = \frac{1}{a_3 - a_2}\log(\frac{u_{20}}{u_{30}})$

respectively and thereafter the dominance is reversed. Also the Host (S_4) of the predator (S_2) dominates over the prey (S_1) till the time instant $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ and thereafter the dominance is reversed as shown in Fig. 6.2.

Case (iii): *If* $u_{10} < u_{40} < u_{30} < u_{20}, \quad a_3 < a_1 < a_2 < a_4$

In this case the prey (S_1) has the least initial population strength and the host (S_3) of the prey (S_1) has the least natural growth rate. The predator (S_2) initially dominates over the host (S_3) of the prey (S_1) and also the prey (S_1) till the time instant $t_{32}^* = \frac{1}{a_3 - a_2}\log(\frac{u_{20}}{u_{30}})$, $t_{12}^* = \frac{1}{a_1 - a_2}\log(\frac{u_{20}}{u_{10}})$ respectively and thereafter the dominance is reversed. Also the host (S_4) of the predator (S_2) dominates over the prey (S_1) till the time instant $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ and thereafter the dominance is reversed as shown in Fig. 6.3.

Case (iv): *If* $u_{10} < u_{20} < u_{40} < u_{30}, \quad a_2 < a_3 < a_1 < a_4$

In this case the predator (S_2) has the least natural growth rate and the prey (S_1) has the least initial population strength. The host (S_4) of the predator (S_2) initially dominates over the predator (S_2) till the time instant $t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}})$ and thereafter the dominance is reversed. Also the host (S_3) of the prey (S_1) dominates over the predator (S_2) till the time instant $t_{23}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$ and thereafter the dominance is reversed. Also the host (S_4) of the predator (S_2) dominates over the prey (S_1) till the time instant $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ and thereafter the dominance is reversed as shown in Fig. 6.4.

Case (v): *If* $u_{20} < u_{10} < u_{30} < u_{40}, \quad a_3 < a_2 < a_1 < a_4$

In this case the predator (S_2) has the least initial population strength and the host (S_3) of the prey (S_1) has the least natural growth rate. The host (S_4) of the predator (S_2) initially dominates over the host (S_3) of the prey (S_1), and the predator (S_2), and also the prey (S_1) till the time instant $t_{34}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}})$, $t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}})$ and $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ respectively and thereafter the dominance is reversed.

Also the prey (S_1) dominates over the predator (S_2) till the time instant $t_{21}^* = \frac{1}{a_2 - a_1}\log(\frac{u_{10}}{u_{20}})$ and thereafter the dominance is reversed as shown in Fig. 6.5.

Case (vi): *If* $u_{20} < u_{30} < u_{40} < u_{10}, \quad a_4 < a_2 < a_1 < a_3$

In this case the predator (S_2) has the least initial population strength and host (S_4) of the predator (S_2) has the least natural growth rate. The prey (S_1) initially dominates over the host (S_4) of the predator (S_2), and also over the predator (S_2), till the time instant $t_{41}^* = \frac{1}{a_4 - a_1}\log(\frac{u_{10}}{u_{40}})$ and $t_{21}^* = \frac{1}{a_2 - a_1}\log(\frac{u_{10}}{u_{20}})$ respectively and thereafter the dominance is reversed. Also the host (S_3) of the prey (S_1) dominates over the predator (S_2) till the time instant $t_{23}^* = \frac{1}{a_2 - a_3}\log(\frac{u_{30}}{u_{20}})$ and thereafter the dominance is reversed as shown in Fig.6.6.

Case (vii): *If* $u_{40} < u_{30} < u_{20} < u_{10}, \quad a_2 < a_3 < a_1 < a_4$

In this case the predator (S_2) has the least natural growth rate.
The prey (S_1) initially dominates over the predator (S_2), and also over the host (S_3) of the prey (S_1), till the time instant $t_{21}^* = \frac{1}{a_2 - a_1}\log(\frac{u_{10}}{u_{20}})$ and $t_{31}^* = \frac{1}{a_3 - a_1}\log(\frac{u_{10}}{u_{30}})$ respectively and thereafter the dominance is reversed as shown in Fig. 6.7.

Case (viii): *If* $u_{20} < u_{30} < u_{40} < u_{10}, \quad a_1 < a_4 < a_3 < a_2$

In this case prey (S_1) has the least natural growth rate and the highest initial population strength. And the predator (S_2) has the highest natural growth rate and the least initial population strength as shown in Fig 6.8.

Case (ix): *If* $u_{30} < u_{10} < u_{20} < u_{40}, \quad a_2 < a_3 < a_1 < a_4$

In this case the predator (S_2) has the least natural growth rate. The host (S_4) of the predator (S_2) initially dominates over the predator (S_2) and the host (S_3) of the prey (S_1), and also the prey (S_1) till the time instant $t_{24}^* = \frac{1}{a_2 - a_4}\log(\frac{u_{40}}{u_{20}})$, $t_{34}^* = \frac{1}{a_3 - a_4}\log(\frac{u_{40}}{u_{30}})$ and $t_{14}^* = \frac{1}{a_1 - a_4}\log(\frac{u_{40}}{u_{10}})$ respectively and thereafter the dominance is reversed. Also the prey (S_1) dominates over host (S_3) of the prey (S_1) till the time instant $t_{31}^* = \frac{1}{a_3 - a_1}\log(\frac{u_{10}}{u_{30}})$ and thereafter the dominance is reversed as shown in Fig. 6.9.

Case (x): *If* $u_{30} < u_{20} < u_{10} < u_{40}, \quad a_4 < a_2 < a_3 < a_1$

In this case the prey (S_1) has the highest natural growth rate.
The prey (S_1) initially dominates over the predator (S_2) and also over the host (S_4) of the predator (S_2), till the time instant $t_{21}^* = \frac{1}{a_2 - a_1}\log(\frac{u_{10}}{u_{20}})$, $t_{31}^* = \frac{1}{a_3 - a_1}\log(\frac{u_{10}}{u_{30}})$ respectively and thereafter the dominance is reversed as shown in Fig. 6.10.

8 Trajectories of Perturbations

The trajectories in $u_1 - u_2,\ u_1 - u_3,\ u_1 - u_4,\ u_2 - u_3,\ u_2 - u_4,\ u_3 - u_4$ *planes are*

$$\left(\frac{u_1}{u_{10}}\right)^{a_2} = \left(\frac{u_2}{u_{20}}\right)^{a_1},\ \left(\frac{u_1}{u_{10}}\right)^{a_3} = \left(\frac{u_3}{u_{30}}\right)^{a_1},\ \left(\frac{u_1}{u_{10}}\right)^{a_4} = \left(\frac{u_4}{u_{40}}\right)^{a_1},$$

$$\left(\frac{u_2}{u_{20}}\right)^{a_3} = \left(\frac{u_3}{u_{30}}\right)^{a_2},\ \left(\frac{u_2}{u_{20}}\right)^{a_4} = \left(\frac{u_4}{u_{40}}\right)^{a_2} \text{ and } \left(\frac{u_3}{u_{30}}\right)^{a_4} = \left(\frac{u_4}{u_{40}}\right)^{a_3}$$

respectively and these are illustrated in the following figures from 8.1 to 8.6.

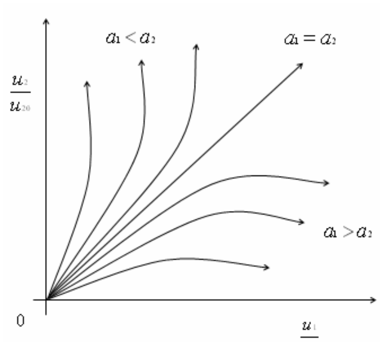

Fig. 8.1

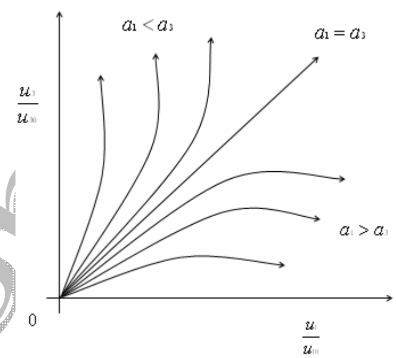

Fig. 8.2

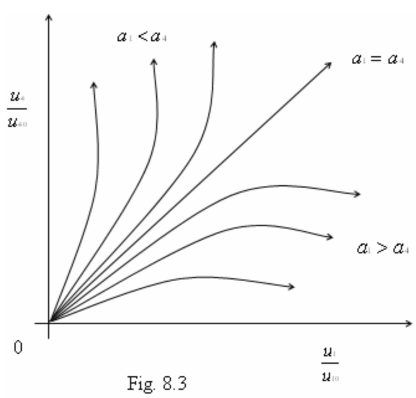

Fig. 8.3

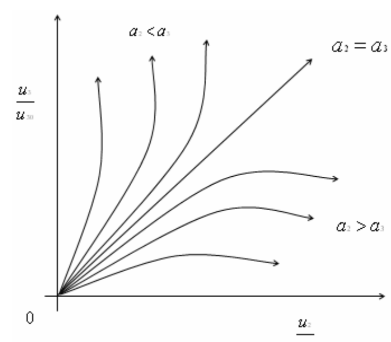

Fig. 8.4

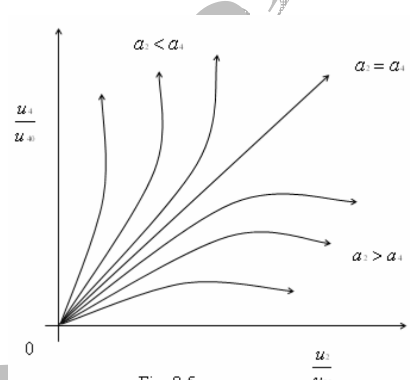

Fig. 8.5

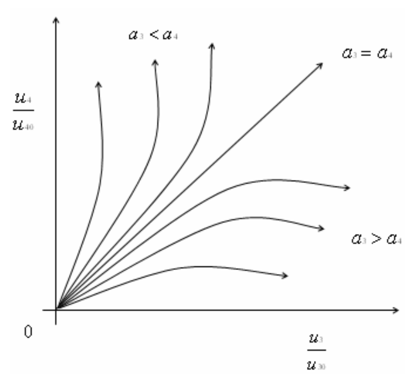

Fig. 8.6

9 Open Problem

Investigate some relation - chains between the species such as Prey-Predation, Competition, Mutualism, Commensalism and Ammensalism between four species N_1, N_2, N_3, N_4 (say) with the population relations

N_1 a prey to N_2 and commensal to N_3
N_2 is a predator living on N_1 and commensal to N_4.
N_3 a host to N_1, N_4 a host to N_2 and N_3, N_4 is mutual.

The present paper deals with the study on stability of fully washed out state only of the above problem. The stability of the other equilibrium states is to be investigated and the perturbation curves are to be studied. The trajectories of perturbations of the other equilibrium states can also be studied.

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