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SORET AND DUFOUR EFFECTS ON UNSTEADY MHD FREE CONVECTIVE MASS TRANSFER FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH OSCILLATORY SUCTION VELOCITY IN THE PRESENCE OF THERMAL RADIATION

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Abstact:

An analysis of free convection and mass transfer unsteady magnetohydrodynamic flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate under oscillatory suction velocity and thermal radiation by taking into account the Dufour (diffusion thermo) and Soret (thermal diffusion) effects. The problem is solved numerically by using finite element method for velocity, temperature and concentration field and also the expression for skin friction, rate of heat and mass transfer have been obtained. The results obtained have been presented numerically through graphs and tables for externally cooled plate ($Gr > 0$) and externally heated plate ($Gr < 0$) to observe the effects of various parameters encountered in the equations.

Key words: Mass transfer, MHD flow, vertical plate, suction velocity, Soret and Dufour numbers, thermal radiation, finite element method.

Introduction:

The range of free convective flows that occur in nature and in engineering practice is very large and have been extensively considered by many researchers (Jaluria, 1980, Gupta *et al.* 1977, among others). When heat and mass transfer occur simultaneously between the fluxes the driving potentials are of more intricate nature. An energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition is called Dufour or diffusion-thermo effect. Temperature gradients can also create mass fluxes, and this is the Soret or thermal-diffusion effect. Generally, the thermal-diffusion and the diffusion-thermo effects are of smaller order magnitude than the effects prescribed by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes.

The study of radiative heat and mass transfer in convective flows is important from many industrial and technological points of view. Mass transfer is one of the most commonly encountered phenomenon in chemical industries as well as in physical and biological sciences. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer process are similar in nature. A number of investigations have already been carried out with combined heat and mass transfer under the assumption of different physical situations. Radiation in free convection has also been studied by many authors because of its applications in many engineering and industrial process. Examples include nuclear power plant, solar power technology, steel industry, fossil fuel combustion, space sciences applications, etc.

In recent years, progress has been considerably made in the study of heat and mass transfer in MHD flows due to its application in many devices, like the MHD power generators and Hall accelerators. Kinyanjui *et al.* (2001) presented simultaneous heat and mass transfer in unsteady free convection flow with radiation absorption past an impulsively started infinite vertical porous plate subjected to a strong magnetic field. Yih (1997) numerically analyzed the effect of transpiration velocity on the heat and mass transfer characteristics of mixed convection about a permeable vertical plate embedded in a saturated porous medium under the coupled effects of thermal and mass diffusion. Elbashbeshy (2003) studied the effect of surface mass flux on mixed convection along a vertical plate embedded in porous medium.

Chin *et al.* (2007) obtained numerical results for the steady mixed convection boundary layer flow over a vertical impermeable surface embedded in a porous medium when the viscosity of the fluid varies inversely as a linear function of the temperature. Pal and Talukdar (2009) analyzed the combined effect of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium is analyzed. Mukhopadhyay (2009) performed an analysis to investigate the effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Hayat *et al.* (2010) analyzed a mathematical model in order to study the heat and mass transfer characteristics in mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a viscoelastic fluid, by taking into account the diffusion thermo (Dufour) and thermal-diffusion (Soret) effects.

Ming-chun *et al.* (2006) took an account of the thermal diffusion and diffusion-thermo effects, to study the properties of the heat and mass transfer in a strongly endothermic chemical reaction system for a porous medium. Gaikwad *et al.* (2007) investigated the onset of double diffusive convection in a two component couple of stress fluid layer with Soret and Dufour effects using both linear and non-linear stability analysis. Osalusi *et al.* (2008) investigated thermo-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and Ohmic heating. Shateyi (2008) investigated thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing.

The interaction of buoyancy with thermal radiation has increased greatly during the last decade due to its importance in many practical applications. The thermal radiation effect is important under many isothermal and non isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then thermal radiation could be important. The knowledge of radiation heat transfer in the system can, perhaps, lead to desired product with sought characteristics. Motsa (2008) investigated the effect of both the Soret and Dufour effects on the onset of double diffusive convection. Mansour *et al.* (2008) investigated the effects of chemical reaction, thermal stratification, Soret and Dufour number on MHD free convective heat and mass transfer of a viscous, incompressible and electrically conducting fluid on a vertical stretching surface embedded in a saturated porous medium. Srihari *et al*. (2006) studied the Soret effect on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with oscillatory suction velocity and heat sink.

Motivated by the above reference work and the numerous possible industrial applications of the problem (like in isotope seperation), it is of paramount interest in this study to investigate the effects of thermal radiation , Soret and Dufour on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with oscillatory suction velocity. None of the above investigations simultaneously studied the effects of Soret and Dufour on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with oscillatory suction velocity in the presence of thermal radiation. Hence, the purpose of this paper is to extend Srihari (2006), to study the more general problem which include Soret and Dufour effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with oscillatory suction velocity in the presence of thermal radiation. In this study the effects of different flow parameters encountered in the equations were also studied. The problem is solved numerically by using finite element method, which is more economical from computational view point.

Mathematical Formulation:

We consider the unsteady hydromagnetic flow of an incompressible, electrically conducting viscous fluid through porous medium, past an infinite vertical plate with oscillatory suction and thermal radiation. In Cartesian co-ordinate system, x'-axis is assumed to be along plate in the direction of the flow and y'-axis normal to it. A normal magnetic field is assumed to be applied in y' – direction and the induced magnetic field is negligible in comparison with the applied one which corresponds to very small magnetic Reynolds number. The surface is maintained at uniform constant temperature and concentration. The flow has significant Thermal

Radiation, Soret and Dufour effects. Under the stated assumptions and taking the usual Boussinesq's approximation in to account.

The governing equations for momentum, energy and concentration in dimensionless form are:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon e^{\text{int}}\right)\frac{\partial u}{\partial y}=Gr\,T+GmC+\frac{\partial^2 u}{\partial y^2}-\frac{u}{K_o\left(1+\varepsilon e^{\text{int}}\right)}-M^2u \qquad (1)$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-(1+\varepsilon e^{\text{int}})\frac{\partial T}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4}{3N}\right)\frac{\partial^2 T}{\partial y^2}+Du\left(\frac{\partial^2 C}{\partial y^2}\right) \qquad (2)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-(1+\varepsilon e^{\text{int}})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+So\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

The relevant boundary conditions in dimensionless form are

$$u=0,\quad T=1+\varepsilon e^{\text{int}},\; C=1+\varepsilon e^{\text{int}} \quad \text{at } y=0$$
$$u\to 0,\quad T\to 0,\quad C\to 0,\quad \text{as } y\to\infty \qquad (4)$$

The non-dimensionless quantities introduced in these equations are defined as:

$$y=\frac{v_o y'}{4v};\; t=\frac{v_o^2 t'}{4v};\; \eta=\frac{4\vartheta\eta'}{v_o^2};\; u=\frac{u'}{v_o};\; T=\frac{T'-T_\infty}{T_w-T_\infty};\; C=\frac{C'-C_\infty}{C_w-C_\infty};$$

$$K_o=\frac{K'_o v_o^2}{\vartheta^2}(Constant\ permeability\ of\ the\ medium);$$

$$So=\frac{D_1(T_w-T_\infty)}{\vartheta(C_w-C_\infty)}(Soret\ number);\; Du=\frac{D_1(C_w-C_\infty)}{\vartheta(T_w-T_\infty)}(Dufour\ number);$$

$$Gr=\frac{\vartheta g\beta^*(T_w-T_\infty)}{v_o^3}(Grashof\ Number);\; Sc=\frac{\vartheta}{D_1}(Schimidt\ Number);$$

$$\Pr=\frac{\mu C_p}{K_t}(\Pr andtl\ Number);\; M=\frac{B_o}{v_o}\sqrt{\frac{\sigma\vartheta}{\rho}}(Magnetic\ Parameter)\ and$$

$$Gm=\frac{\vartheta g\beta(C_w-C_\infty)}{v_o^3}(Modofied\ Grashof\ Number).;\; N=\frac{kk_e}{4\sigma_s T_\infty^3}(Thermal\ Radiation);$$

Where u is the velocity along the x-axis, ρ is the density of the fluid, ϑ is the kinematic coefficient of viscosity, η is the frequency of oscillation, g is the acceleration due to gravity, t is the time, β is the coefficient of volume expansion for the heat transfer, β* is the volumetric coefficient of expansion with species concentration, T is the fluid temperature, T_∞ is the fluid temperature at infinity, C is the species concentration, C_∞ is the species concentration at infinity, D_1 is the chemical molecular diffusivity, Ko is the constant permeability of the medium, μ is the coefficient of viscosity, Cp is the specific heat at constant pressure.

Method of Solution:

The Galerkin equation for the differential equation (1) becomes

$$\int_{y_j}^{y_k} \left\{ N^T \left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{1}{4}\frac{\partial u^{(e)}}{\partial t} + A\frac{\partial u^{(e)}}{\partial y} - Ru^{(e)} + P \right] \right\} dy = 0$$

Where $N^T = \begin{bmatrix} N_j & N_k \end{bmatrix}^T = \begin{bmatrix} N_j \\ N_k \end{bmatrix}$, A $=1+\varepsilon e^{\text{int}}$, R $= \dfrac{1}{K_o A} + M^2$, P $= GrT + Gm\ C$

Let the linear piecewise approximation solution

$$u^{(e)} = N_j(y)u_j(t) + N_k(y)u_k(t) = N_j u_j + N_k u_k$$

The element equation is given by

$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \frac{1}{4}\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy - A\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy$$

$$+R\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy - P\int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy = 0$$

Where prime and dot denotes differentiation w.r.to 'y' and ' t ' respectively.

Simplifying we get

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{A}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{R}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where $l^{(e)} = y_k - y_j = h$

In order to get the differential equation at the knot x_i, we write the element equations for the elements $y_{i-1} \leq y \leq y_i$ and $y_i \leq y \leq y_{i+1}$ assemble three element equations, we obtain

$$\frac{1}{l^{(e)^2}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{24}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{A}{2l^{(e)}}\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{R}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix}$$

$$= \frac{P}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

We put the row equation corresponding to the knot 'i', is

$$\frac{1}{l^{(e)^2}}\left[-u_{i-1} + 2u_i - u_{i+1}\right] + \frac{1}{24}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] - \frac{A}{2l^{(e)}}\left[-u_{i-1} + u_{i+1}\right] + \frac{R}{6}\left[u_{i-1} + 4u_i + u_{i+1}\right] = P \qquad (5)$$

Applying Crank-Nicholson method to the above equation (5), then we gets

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = B_1 u_{i-1}^{n} + B_2 u_i^{n} + B_3 u_{i+1}^{n} + P^* \qquad (6)$$

Where $A_1 = 1 + 2Rk - 12r + 6Arh$; $A_2 = 4 + 24r + 8Rk$; $A_3 = 1 + 2Rk - 12r - 6Arh$;

$B_1 = 1 - 2Rk + 12r - 6Arh$; $B_2 = 4 - 24r - 8Rk$; $B_3 = 1 - 2Rk + 12r + 6Arh$;

$P^* = 24(Gr)kT_i^{\,j} + 24(Gm)kC_i^{\,j}$;

Applying similar procedure to equation (2), then we gets

$$G_1 T_{i-1}^{n+1} + G_2 T_i^{n+1} + G_3 T_{i+1}^{n+1} = H_1 T_{i-1}^{n} + H_2 T_i^{n} + H_3 T_{i+1}^{n} + Q^* \qquad (7)$$

Where $G_1 = Pr + 6Arh(Pr) - 12Dr$; $G_2 = 4Pr + 24Dr$; $G_3 = Pr - 6Arh(Pr) - 12Dr$;

$H_1 = Pr - 6Arh(Pr) + 12Dr$; $H_2 = 4Pr - 24Dr$; $H_3 = Pr + 6Arh(Pr) + 12Dr$;

$$D = 1 + \left(\frac{4}{3N}\right); \; Q^* = 24k(\mathrm{Pr})Du\left(\frac{\partial^2 C}{\partial y^2}\right);$$

Applying similar procedure to equation (3), then we gets

$$J_1 C_{i-1}^{n+1} + J_2 C_i^{n+1} + J_3 C_{i+1}^{n+1} = I_1 C_{i-1}^{n} + I_2 C_i^{n} + I_3 C_{i+1}^{n} + B^* \qquad (8)$$

Where $J_1 = Sc + 6Arh(Sc) - 12r$; $J_2 = 4Sc + 24r$; $J_3 = Sc - 6Arh(Sc) - 12r$;

$I_1 = Sc - 6Arh(Sc) + 12r$; $I_2 = 4Sc - 24r$; $I_3 = Sc + 6Arh(Sc) + 12r$;

$$B^* = 24k(Sc)So\left(\frac{\partial^2 T}{\partial y^2}\right);$$

Here $r = \frac{k}{h^2}$ and k, h are mesh sizes along y- direction and time-direction respectively index 'i' refers to space and j refers to time .The mesh system consists of h=0.1 and k=0.01.

In the equations (6), (7) and (8), taking i = 1(1)10 and using boundary conditions in (4), then we get the following tri-diagonal system of equations

$$Au = B \quad (9)$$

$$ET = F \quad (10)$$

$$XC = Y \quad (11)$$

Where A, E and X are the tri-diagonal matrices of order ten and whose elements are defined by

$A_{i,i} = A_2$; $E_{i,i} = G_2$; $X_{i,i} = J_2$; at i = 1(1) n

$A_{i-1,i} = A_1$; $E_{i-1,i} = G_1$; $X_{i-1,i} = J_1$; at i = 2(1) n

$A_{i,i+1} = A_3$; $E_{i,i+1} = G_3$; $X_{i,i+1} = J_3$; at i = 2(1) n

And u, B, T, F, C, Y are column matrices having n-components, which are column matrices having components namely $u_{i,j+1}$, B(i,j), $T_{i,j+1}$, F(i,j), $C_{i,j+1}$ and Y(i,j); i=1(1)10 respectively. The solutions of (9), (10) and (11) can be obtained by Thomas algorithm. To judge the accuracy of convergence and stability of finite element method, the programmed was run with smaller values of k. i.e. k=0.005 and no significant change was observed. Hence we conclude that the finite element method is stable and convergent.

Skin-friction, Rate of heat and mass transfer:

Skin-Friction co-efficient (τ) at the plate is $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$

Heat transfer co-efficient (N_u) at the plate is $N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$

Mass transfer co-efficient (S_b) at the plate is $S_b = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$

RESULTS AND DISCUSSIONS:

Some numerical calculations have been carried out for the non-dimensional velocity (u), temperature (T), concentration (C), skin-friction coefficient (τ) and heat and mass transfer coefficients in terms of Nusselt number (Nu) and Sherwood number (Sh) respectively. The effects of material parameters such as Prandtl number (Pr), Schmidt number (Sc), magnetic parameter (M), permeability parameter (Ko), Soret number or Thermal Diffusion (So), Dufour number or Diffusion Thermo (Du), Radiation parameter (N), Grashof number (Gr) and modified Grashof number (Gm) have been observed. The numerical calculations of these results are presented graphically in Figs 1 to 12. During the course of numerical calculations of the velocity, temperature and concentration, the values of the Prandtl number are chosen for air (Pr = 0.71), electrolytic solution (Pr = 1.0), water (Pr = 7.0) and water at 4^oC (Pr = 11.40). To focus out attention on numerical values of the results obtained in the study the values of Sc are chosen for the gases representing diffusing chemical species of most common interest in air namely Hydrogen (Sc = 0.22), Water-vapour (Sc = 0.60), Oxygen (Sc = 0.66), Methanol (Sc = 1.00) and Propyl-benzene (Sc = 2.62) at 20^oC and one atmospheric pressure. For the physical significance, only the real part of complex quantity is invoked for the numerical discussion in the problem and at t=1.0, stable values for velocity, temperature and concentration fields are obtained. To examine the effect of parameters related to the problem on the velocity field and skin-friction numerical computations are carried out at (Pr = 0.71) which corresponds to air at 25^oC and one atmospheric pressure. The values of Grashof number Gr and modified Grashof number Gm are taken to be positive and negative as they respectively represent symmetric cooling of the channel walls when Gr > 0 and symmetric heating of the channel walls when Gr < 0.

The temperature and the species concentration are coupled to the velocity via Grashof number Gr and modified Grashof number Gm as seen in equation (1). Figures $1-7$ display the effects of material parameters such as Gr, Gm, M, N, Du, So and Ko on the velocity field for both externally cooling (Gr > 0) and heating (Gr < 0) of the plate. It is observed that an increase in the Grashof number or modified Grashof number leads to increase in the velocity field in both the presence of cooling and heating of the plate. A comparison of velocity field curves due to cooling of the plate shows that the velocity increases rapidly near the plate and after attaining a maximum value, it decreases as y increases.

The effect of Magnetic field parameter M is shown in the figure 6 in case of cooling of the plate. It is observed that the velocity of the fluid decreases with the increase of Magnetic parameter values. We also see that velocity profiles decrease with the increase of magnetic effect indicating that magnetic field tends to retard the motion of the fluid. Magnetic field may control the flow characteristics.

The influence of thermal radiation on the velocity is shown on figure 5 in case of cooling of the plate. Increasing thermal radiation parameter produces a decrease in the velocity of the flow. It is observed that the velocity decreases in the presence of higher thermal radiation.

Figures (3) and (4) show the effects of Dufour number Du and Soret number So on velocity field u. From these figures, it is observed that the velocity u increases as Dufour number Du and Soret number So increases in case of cooling of the plate. It is interesting note that the effect of Dufour and Soret numbers on velocity field are significant. Figure (7) exhibits the effect of Ko on velocity field u. From this figure, it is observed that an increase in Ko increases the velocity u in case of cooling of the plate. In the figures (3) – (7) on velocity field mentioned above, compare to the case of cooling of the plate opposite effects are observed in the case of heating of the plate.

In figure (8) we depict the effect of Prandtl number Pr on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. Also, temperature field falls more rapidly for water in comparison to air and the temperature curve is exactly linear for mercury, which is more sensible towards change in temperature. From this observation it is conclude that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Air can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. If temperatures are maintained air can be better and cheap replacement for industrial purpose. Figure (9) shows the plot of temperature of the flow field against for three different values of radiation parameter N taking other parameters are constant. The radiation parameter is found to decelerate the temperature of the flow field at all points. Higher the radiation parameter, the more sharper is the reduction in the temperature. Figure (10) depicts the effects of the Dufour number on the fluid temperature. It can be clearly seen from this figure that diffusion thermal effects greatly affect the fluid temperature. As the values of the Dufour number increase, the fluid temperature also increases.

The effects of Schmidt number Sc and Soret number So on the concentration field are presented in figures (11) and (12). In the figures, it is observed that an increase in the Schmidt number leads to decrease in the concentration field while an increase in the Soret number leads to increase in the concentration field.

Table -1 presents numerical values of the skin-friction coefficient (τ) for variations in Gr, Gm, M, Ko, Pr, Sc, So, Du and N for externally cooling of the plate (Gr > 0). It is observed that, an increase in the Prandtl number or Schmidt number or magnetic parameter or radiation parameter leads to decrease in the value of skin-friction coefficient while an increase in the permeability parameter or Soret number or Dufour number or Grashof number or modified Grashof number leads to increase in the value of skin-friction coefficient.

Table - 2 presents numerical values of the skin-friction coefficient (τ) for variations in Gr, Gm, M, Ko, Pr, Sc, So, Du and N for externally heating of the plate (Gr < 0). It is observed that, an increase in the Schmidt number or permeability parameter or Dufour number leads to decrease in the value of skin-friction coefficient while an increase in the Prandtl number or magnetic parameter or Soret number or Grashof number or modified Grashof number leads to increase in the value of skin-friction coefficient.

Table - 3 presents numerical values of heat transfer coefficient in terms of Nusselt number (Nu) for different values of the Prandtl number Pr, Radiation parameter N and Dufour number Du respectively. It is observed that an increase in the Dufour number leads to decrease in the value of heat transfer coefficient while an increase in the Prandtl number or radiation parameter leads to decrease in the value of heat transfer coefficient.

Table - 4 presents numerical values of mass transfer coefficient in terms of Sherwood number (Sh) for different values of Schmidt number Sc, Radiation parameter N and Soret number So respectively. It is observed that an increase in the Schmidt number leads to decrease in the value of mass transfer coefficient while an increase in the Soret number and Radiation parameter leads to increase in the value of mass transfer coefficient.

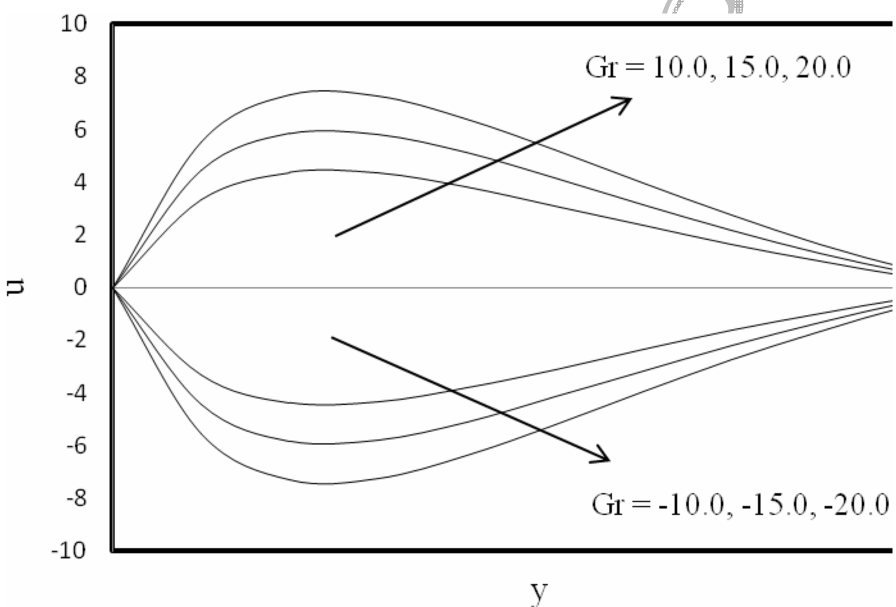

Figure 1. Effect of Heat transfer ‘Gr’ on Velocity field ‘u’ for cooling (Gm = 5.0) and heating of the plate (Gm = -5.0) when M = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, So = 1.0, Du = 0.03, N = 0.5, ε = 0.005 and nt = π/2.

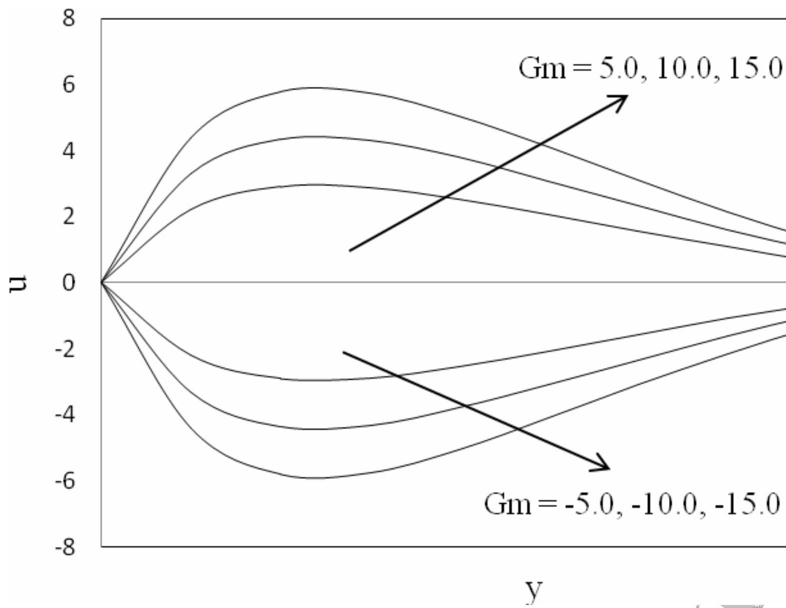

Figure 2. Effect of Mass transfer ‘Gm’ on Velocity field ‘u’ for cooling (Gr = 5.0) and heating of the plate (Gr = -5.0) when M = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, So = 1.0, Du = 0.03, N = 0.5, ε = 0.005 and nt = π/2.

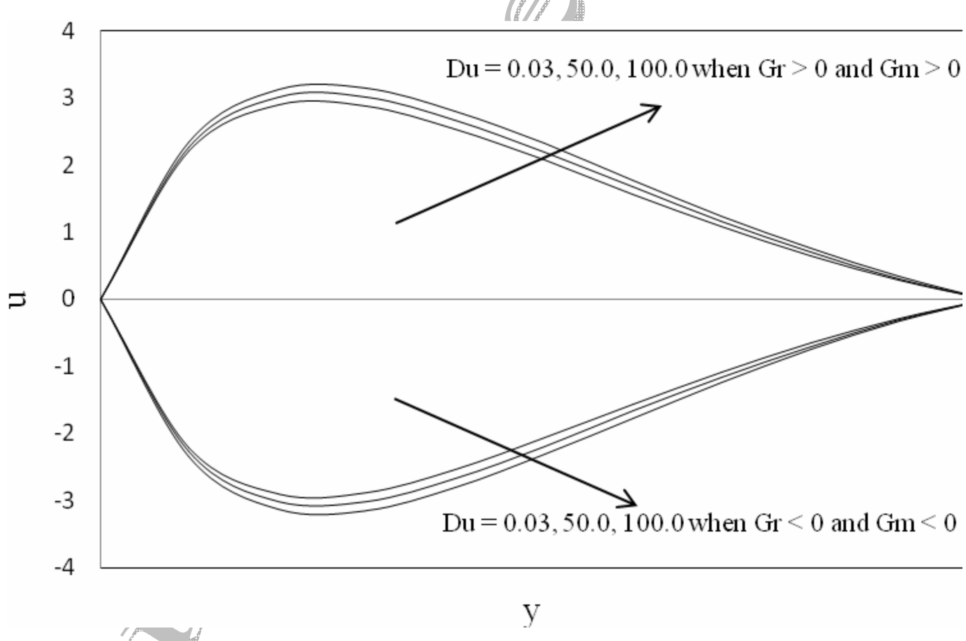

Figure 3. Effect of Dofour number ‘Du’ on Velocity field ‘u’ for cooling (Gr = 5.0 & Gm = 5.0) and heating of the plate (Gr = -5.0 & Gm = -5.0) when M = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, So = 1.0, N = 0.5, ε = 0.005 and nt = π/2.

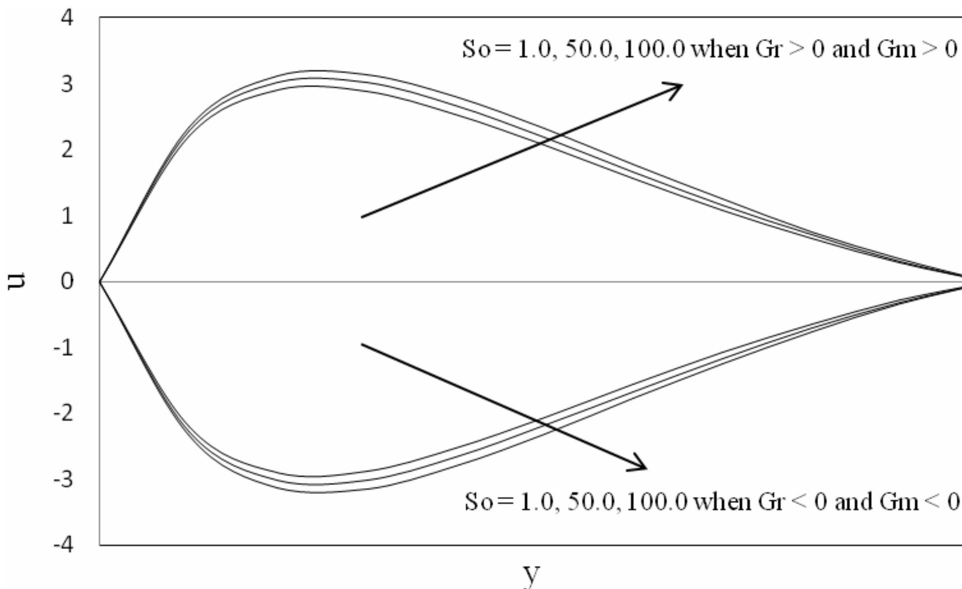

Figure 4. Effect of Soret number ‘So’ on Velocity field ‘u’ for cooling (Gr = 5.0 & Gm = 5.0) and heating of the plate (Gr = -5.0 & Gm = -5.0) when M = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, Du = 0.03, N = 0.5, $\varepsilon = 0.005$ and nt = $\pi/2$.

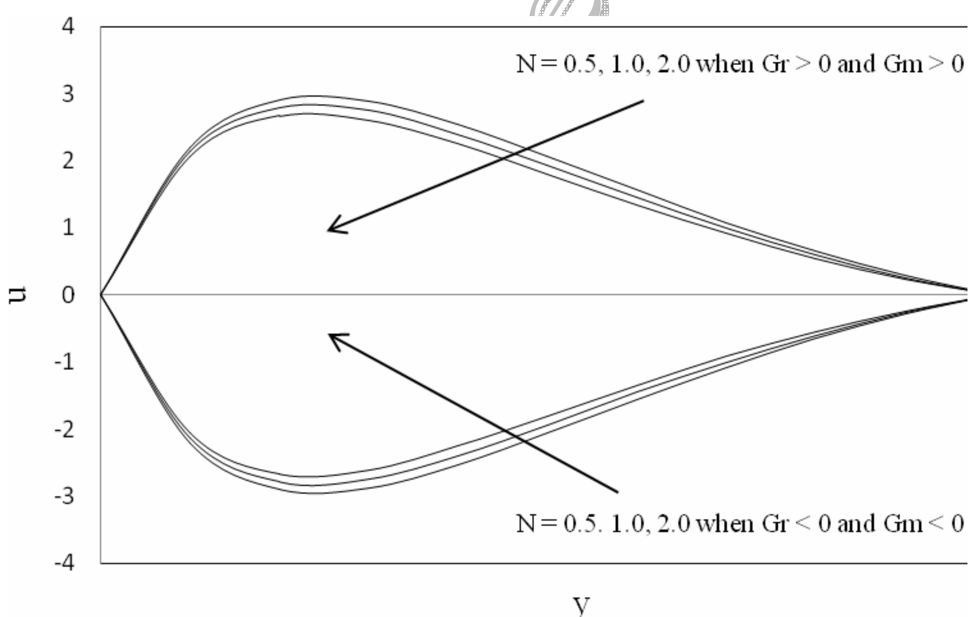

Figure 5. Effect of Thermal radiation parameter ‘N’ on Velocity field ‘u’ for cooling (Gr = 5.0 & Gm = 5.0) and heating of the plate (Gr = -5.0 & Gm = -5.0) when M = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, Du = 0.03, $\varepsilon = 0.005$ and nt = $\pi/2$.

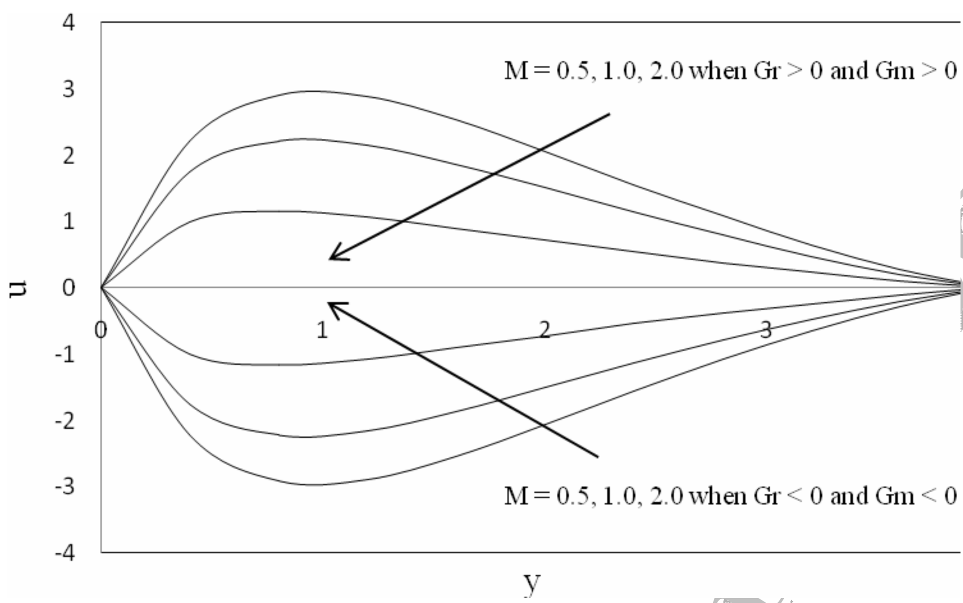

Figure 6. Effect of Magnetic parameter ‘M’ on Velocity field ‘u’ for cooling (Gr = 5.0 & Gm = 5.0) and heating of the plate (Gr = -5.0 & Gm = -5.0) when N = 0.5, Sc = 0.22, Pr = 0.71, Ko = 1.0, Du = 0.03, ε = 0.005 and nt = π/2.

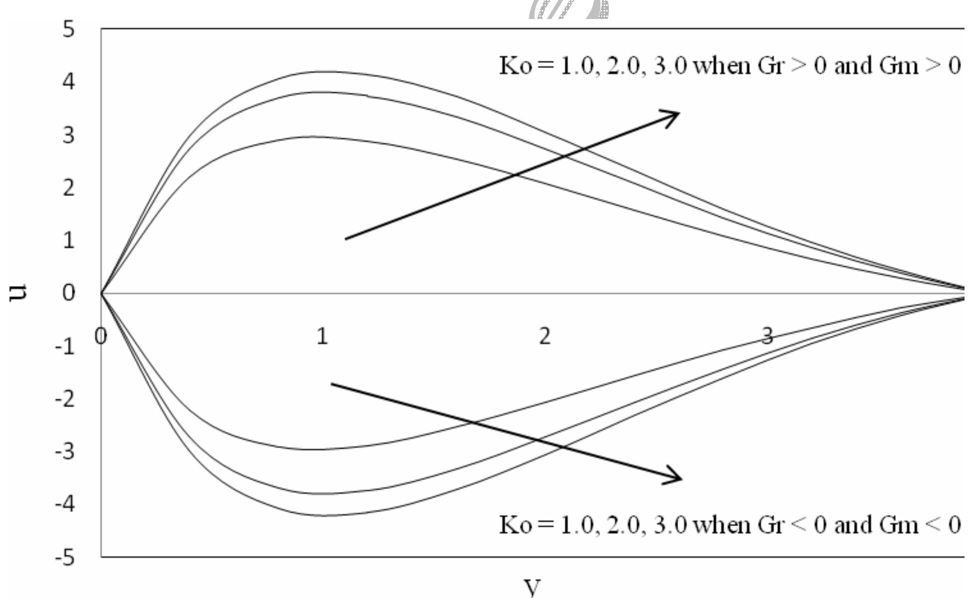

Figure 7. Effect of Permeability parameter ‘Ko’ on Velocity field ‘u’ for cooling (Gr = 5.0 & Gm = 5.0) and heating of the plate (Gr = -5.0 & Gm = -5.0) when N = 0.5, Sc = 0.22, Pr = 0.71, M = 0.5, Du = 0.03, ε = 0.005 and nt = π/2.

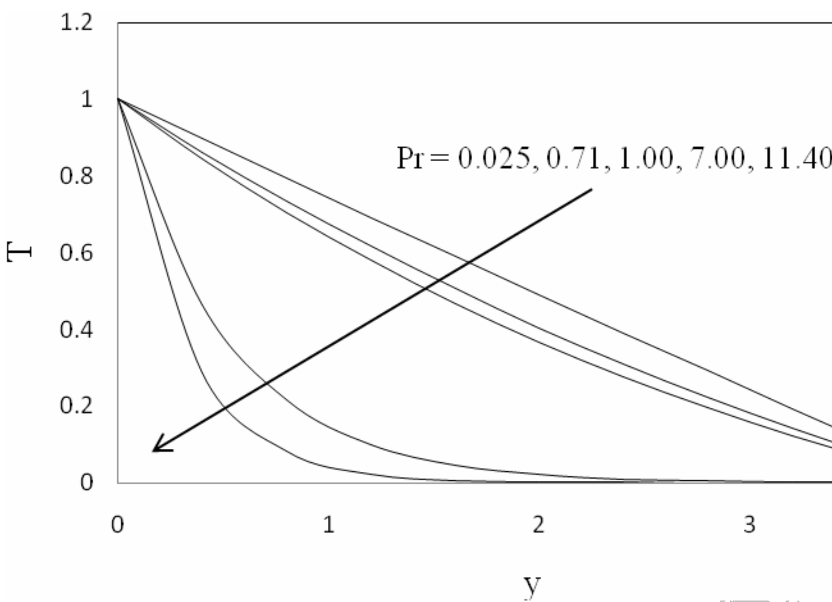

Figure 8. Effect of Prandtl number ‘Pr’ on Temperature field ‘T’ when Gr = 5.0, Gm = 5.0, N = 0.5, Sc = 0.22, Ko = 1.0, M = 0.5, Du = 0.03, ε = 0.005 and nt = π/2.

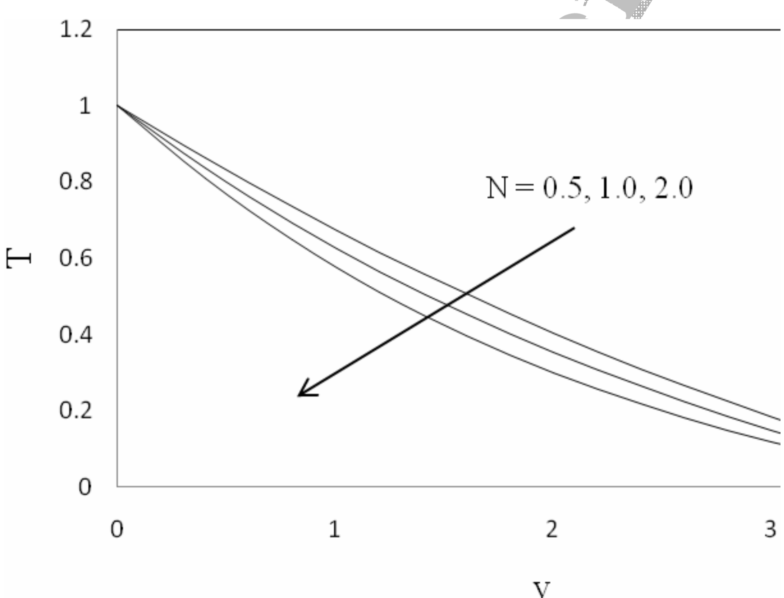

Figure 9. Effect of Thermal radiation parameter ‘N’ on Temperature field ‘T’ when Gr = 5.0, Gm = 5.0, Pr = 0.71, Sc = 0.22, Ko = 1.0, M = 0.5, Du = 0.03, ε = 0.005 and nt = π/2.

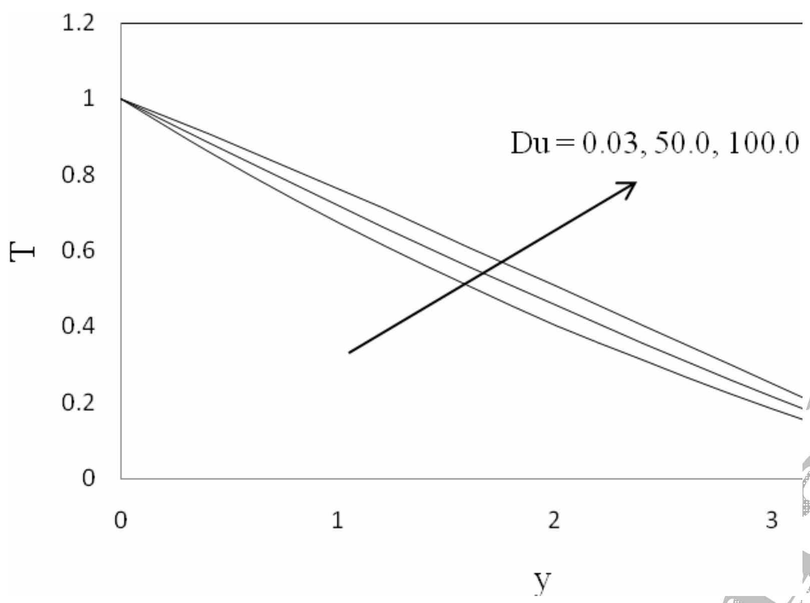

Figure 10. Effect of Dufour number ‘Du’ on Temperature field ‘T’ when Gr = 5.0, Gm = 5.0, Pr = 0.71, Sc = 0.22, Ko = 1.0, M = 0.5, N = 0.5, ε = 0.005 and nt = π/2.

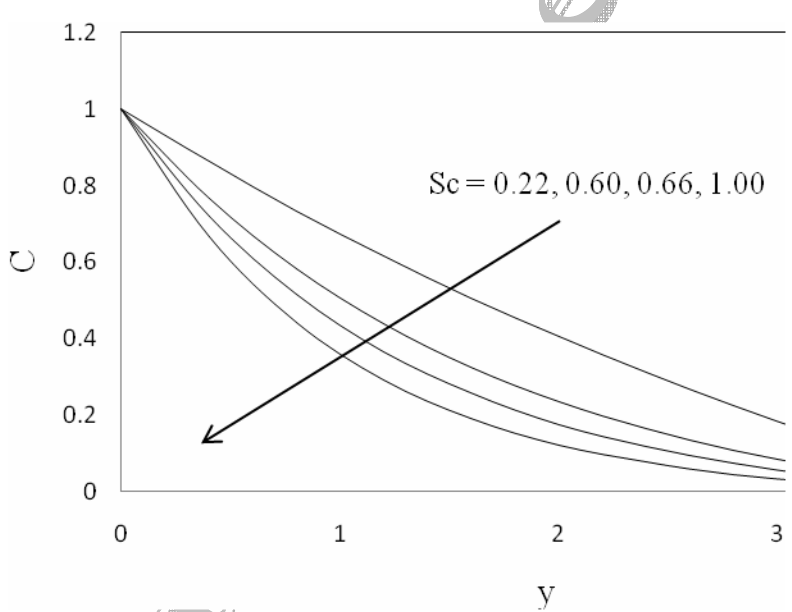

Figure 11. Effect of Schmidt number ‘Sc’ on Concentration field ‘C’ when Gr = 5.0, Gm = 5.0, Pr = 0.71, Du = 0.03, Ko = 1.0, M = 0.5, N = 0.5, ε = 0.005 and nt = π/2.

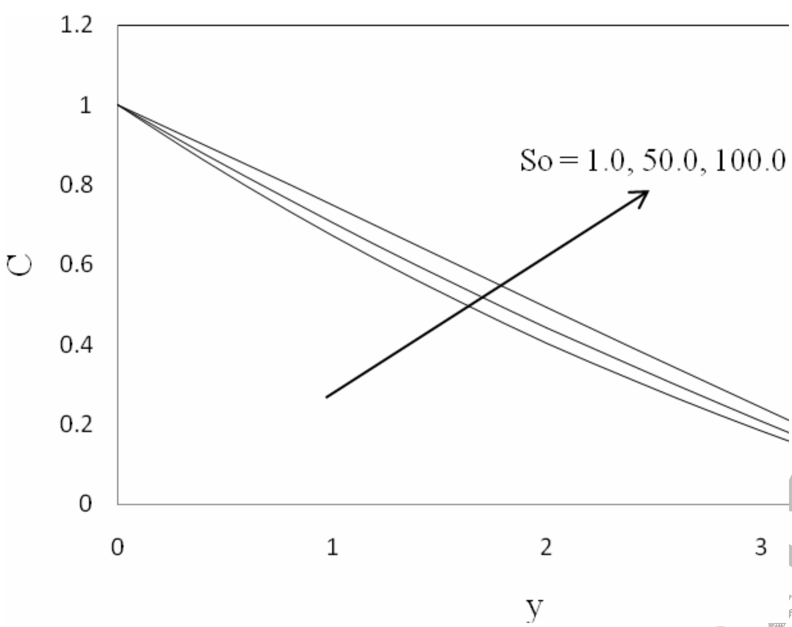

Figure 12. Effect of Soret number 'So' on Concentration field 'C' when Gr = 5.0, Gm = 5.0, Pr = 0.71, Sc = 0.22, Du = 0.03, Ko = 1.0, M = 0.5, N = 0.5, ε = 0.005 and nt = π/2.

Table 1

Skin-friction coefficient (τ) for cooling of the plate

Gr	Gm	M	Ko	Pr	Sc	So	Du	N	τ
5.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	7.6014
10.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	11.4250
5.0	10.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	11.3793
5.0	5.0	1.0	1.0	0.71	0.22	1.0	0.03	0.5	6.1951
5.0	5.0	0.5	2.0	0.71	0.22	1.0	0.03	0.5	9.1764
5.0	5.0	0.5	1.0	7.0	0.22	1.0	0.03	0.5	5.3787
5.0	5.0	0.5	1.0	0.71	0.60	1.0	0.03	0.5	6.9138
5.0	5.0	0.5	1.0	0.71	0.22	2.0	0.03	0.5	7.6054
5.0	5.0	0.5	1.0	0.71	0.22	1.0	1.0	0.5	7.6052
5.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	1.0	7.3959

Table 2

Skin-friction coefficient (τ) for heating of the plate

Gr	Gm	M	Ko	Pr	Sc	So	Du	N	τ
-5.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	-0.0456
-10.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	-3.8692
-5.0	10.0	0.5	1.0	0.71	0.22	1.0	0.03	0.5	3.7323
-5.0	5.0	1.0	1.0	0.71	0.22	1.0	0.03	0.5	-0.0324
-5.0	5.0	0.5	2.0	0.71	0.22	1.0	0.03	0.5	-0.0619
-5.0	5.0	0.5	1.0	7.0	0.22	1.0	0.03	0.5	2.2237
-5.0	5.0	0.5	1.0	0.71	0.60	1.0	0.03	0.5	-0.7337
-5.0	5.0	0.5	1.0	0.71	0.22	2.0	0.03	0.5	-0.0416
-5.0	5.0	0.5	1.0	0.71	0.22	1.0	1.0	0.5	-0.0495
-5.0	5.0	0.5	1.0	0.71	0.22	1.0	0.03	1.0	0.1645

Table 3

Heat transfer coefficient in terms of Nusselt number

Pr	Du	N	Nu
0.71	0.03	0.5	9.6443
7.00	0.03	0.5	8.2569
0.71	1.0	0.5	9.6456
0.71	0.03	1.0	9.5743

Table 4

Mass transfer coefficient in terms of Sherwood number

Sc	So	N	Sh
0.22	1.0	0.5	9.6295
0.60	1.0	0.5	9.3635
0.22	2.0	0.5	9.6308
0.22	1.0	1.0	9.6304

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