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ESTIMATING MEAN OF A NORMAL POPULATION USING KNOWN COEFFICIENT OF VARIATION

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ABSTRACT

In estimating the parameters of a population if one can sacrifice the property of unbiasedness, better estimators in view of minimum mean squared error(MMSE) can be obtained.Consider a normal population N ($\mu, c\mu^2$) with mean μ and known coefficient of variation $\sqrt{c}$ .Since the variance $c\mu^2$ is a function of μ, it is appropriate if s is also considered along with the sample mean $\bar{y}$ for estimating μ. Motivated by this observation, two estimators for μ are proposed in this paper using $\bar{y}$ and s. The procedure adopted is based on that of Searle.D.T (1964). The large sample properties of these estimators and their comparison with that of the conventional estimator, i.e., sample mean $\bar{y}$, are also investigated. The corresponding results are presented. Larger gains are observed in efficiencies for small sample sizes.

1.1 INTRODUCTION

Consider a normal population N ($\mu, c\mu^2$) with mean μ and known coefficient of variation $\sqrt{c}$. Let $y_1, y_2, ...y_n$ be sample of size n from above population.

Define $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ (1.1.1)

$$s^2 = \frac{1}{n-1}\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{Y})^2 \quad (1.1.2)$$

Since the variance $c\mu^2$ is a function of μ, it is appropriate if s is also considered along with the sample mean $\bar{Y}$ for estimating μ. Motivated by this observation, two estimators for μ are proposed below using $\bar{Y}$ and s. the large sample properties of these estimators and their comparison with that of the conventional estimator, i.e., sample mean $\bar{Y}$ are also investigated. The corresponding results are given below.

1.2 FIRST ESTIMATOR FOR MEAN OF N ($\mu, c\mu^2$)

It is well known that $\bar{Y} \sim N(\mu, \frac{c\mu^2}{n})$ (1.2.1)

and for large n, n-1 $\cong$ n, $\frac{\sqrt{2n}}{\mu\sqrt{c}} s \sim N(\sqrt{2n}, 1)$

Consider $t = k(\bar{Y} + s)$ (1.2.2)

As an estimator for μ, where k is a scalar to be determined to minimize the MSE(t). expressions for bias in t, [B(t)] and minimum MSE(t) are derived below to $0(n^{-1})$.

Bias (t) = B (t) = E (t) - μ

B (t) = μ [k ($\sqrt{c}$ + 1) – 1] (1.2.3)

MSE (t) = V (t) + [B (t)]^2 (1.2.4)

But $V(t) = k^2[\frac{c\mu^2}{n} + \frac{c\mu^2}{2n}] = \frac{3k^2\mu^2 c}{2n}$ (1.2.5)

From (1.2.3) and (1.2.4) $MSE(t) = \frac{3k^2\mu^2 c}{2n} + \mu^2[k(\sqrt{c}+1) - 1]^2$ (1.2.6)

Differentiating (1.2.6) with respect to k and equating to zero,

$$k = \frac{1}{\sqrt{c}+1}\left[1 - \frac{3c}{2n(\sqrt{c}+1)^2}\right] \quad (1.2.7)$$

It can be noticed that the second derivation is positive.

Using (1.2.7) in (1.2.2) the estimator is $t = \frac{1}{\sqrt{c}+1}(1 - \frac{3c}{2n(\sqrt{c}+1)^2})(\bar{Y} + S)$ (1.2.8)

Bias in (t) is, $B(t) = \frac{-3c}{2n(\sqrt{c}+1)^2} \cdot \mu$ (1.2.9)

And minium MSE (t) is, $M(t) = \frac{3c\mu^2}{2n(\sqrt{c}+1)^2}$ (1.2.10)

Comparison of t with $\bar{Y}$:

From (1.2.5) and (1.2.10), relative efficiency of t over $\bar{Y}$ is ,

$$REF(t, \bar{Y}) = \frac{V(\bar{Y})}{M(t)} = \frac{2(\sqrt{c}+1)^2}{3} \quad (1.2.11)$$

It can be noticed that t is more efficient than $\bar{Y}$ for $\sqrt{c} > 0.2247$. The values of $REF(t, \bar{Y})$ for specified values of $\sqrt{c}$ are tabulated below.

TABLE 1

$REF(t,\bar{Y})$ in %

$\sqrt{c}$	0.5	1.5	2.5	3.5	4.0
$REF(t,\bar{Y})$	150.00	416.66	816.66	1350.00	2016.66

It can be observed that efficiency of t increases rapidly with increase in c.v.

1.3 SECOND ESTIMATOR FOR MEAN OF N ($\mu, c\mu^2$)

Consider $t' = a\bar{Y} + (1-a)s$ (1.3.1)

As an estimator for μ, where **a** is to be determined such that t' has minimum MSE. Expressions for bias $B(t')$ and $MSE(t')$ $(M(t'))$ are derived to 0 (n^{-1}) below

$$B(t') = E(t') - \mu = (1-a)\mu(\sqrt{c}-1) \quad (1.3.2)$$

$$MSE(t') = V(t') + [B(t')]^2 \quad (1.3.3)$$

But, $$V(t') = \frac{a^2 c\mu^2}{n} + (1-a)^2 \frac{c\mu^2}{2n} \quad (1.3.4)$$

From (1.3.2), (1.3.3) and (1.3.4)

$$MSE(t') = \frac{a^2\mu^2 c}{n} + (1-a)^2 \frac{c\mu^2}{2n} + (1-a)^2 \mu^2 (\sqrt{c}-1)^2 \quad (1.3.5)$$

Differentiating $MSE(t')$ with respect to **a** and equating to zero, $a = 1 - \frac{5c}{4mn}$ (1.3.6)

where $(\sqrt{c}-1)^2 = m$; $\sqrt{c} \neq 1$ (1.3.7)

It can be observed that the second derivative is positive.

Using (1.3.6) in (1.3.1) $t' = (1 - \frac{5c}{4mn})\bar{Y} + \frac{5c}{4mn} s$ (1.3.8)

$$B(t') = \frac{5c\mu}{49\sqrt{c}-1)n} \quad (1.3.9)$$

and minimum mean squared error $M(t')$ is, $M(t') = \frac{c\mu^2}{n}$ (1.3.10)

Comparision of t' and $\overline{Y}$:
From (1.2.1) and (1.3.10)

Relative efficiency of t' over $\overline{Y}$, $REF(t',Y)$ is $REF(t',\overline{Y}) = \dfrac{V(\overline{Y})}{M(t')} = 1$

Hence t' and $\overline{Y}$ are equally efficient.

Remark: It can be seen that $REF(t,t') = \dfrac{M(t')}{M(t)} = REF(t,\overline{Y})$.

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