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INVERSE COEFFICIENT CONDITIONS FOR $S_P(\alpha)$, S_P AND *UCV*

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Abstract:
A normalized function *f* analytic in the open unit disc around the origin and nonvanishing outside the origin can be expressed in the form *z/g(z)* where *g(z)* has Taylor coefficients b_n's. Coefficient conditions in terms of b_n's are derived for functions in the classes $S_{p}(\alpha)$, S_p and *UCV* of univalent analytic functions.

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Introduction: Let A_1 be the class of functions f analytic in $U = \{ z \in \mathbf{C}: |z| < 1\}$, and normalized by $f(0)=0$, $f'(0)=1$ where $\mathbf{C}$ is the set of complex numbers. An f in A_1 with $f(z) \neq 0$ in the punctured disc $U \setminus \{0\}$, may be expressed as $f(z) = \psi(g) = z / g(z)$ in U, where $g(z) = 1 + \sum_{n=1}^{\infty} b_n z^n$ in U. We call b_n's , the inverse coefficients of f.

Mitrinovic [1], Reade et.al [2] ,Silverman and Silvia[5] and Srinivas[6,7] studied these coefficients b_n's.

Mitrinovic [1] obtained estimates for the radius of univalence of certain rational functions. In particular, he found sufficient conditions for functions of the form

$$\frac{z}{1 + b_1 z + b_2 z^2 + \ldots + b_n z^n},$$

$b_n \neq 0$, to be univalent in the unit disk U.

A function

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

in A_1 is said to be in the class *CV* if and only if $f(z)$ is one to one and $f(U)$ is convex. A function $f(z)$ is said to be uniformly convex in U if and only if $f(z)$ is in *CV* and has the property that for every circular arc γ contained in U, with centre ξ also in U, the arc $f(\gamma)$ is convex. The class of uniformly convex functions is denoted by *UCV*. We have

$$f \in UCV \Leftrightarrow \operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \left|\frac{zf''(z)}{f'(z)}\right|, z \in U.$$

Ronning [3] introduced a new class of starlike functions related to *UCV* defined as

$$f \in S_p \Leftrightarrow \left|\frac{zf'(z)}{f(z)} - 1\right| < \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}, z \in U.$$

We have that for a function $f(z)$ in A_1 ,

$$f(z) \in UCV \Leftrightarrow zf'(z) \in S_p$$

Further, Ronning generalized the class S_p by introducing a parameter $\alpha, -1 \leq \alpha < 1$ and defined that for a function $f(z)$ in A_1 ,

$$f \in S_p(\alpha) \Leftrightarrow \left|\frac{zf'(z)}{f(z)} - 1\right| < \operatorname{Re}\left\{\frac{zf'(z)}{f(z)} - \alpha\right\}, z \in U.$$

We have $S_p(0) = S_p$.

Ronning [3] derived necessary and sufficient conditions for a binomial to be in S_p or *UCV*:

Theorem A. $f(z) = z+a_n z^n$ is in S_p if and only if $|a_n| \leq 1/(2n-1)$.

TheoeremB. $f(z) = z+a_n z^n$ is in *UCV* if and only if $|a_n| \leq 1/(n(2n-1))$.

In this paper we derive some sufficient conditions on b_n's for f to be in class $S_p(\alpha)$ in Section1. In Section 2 , we find some necessary and sufficient conditions for some binomials to be in S_p or *UCV*.

Section-1

First we determine a sufficient condition on f in terms of b_n's for f to be $S_p(\alpha)$.

Theorem 1. Let $f(z) = z/(1+\sum_{n=1}^{\infty} b_n z^n) \in A_1$ with b_n's satisfying

$$\sum_{n=1}^{\infty}[2n+(1-\alpha)]|b_n| < 1-\alpha.$$

Then $f(z)$ is in the class $S_p(\alpha)$.

Proof: For $f(z)= z/g(z)$ where $g(z)=1+\sum_{n=1}^{\infty} b_n z^n, z\in U,$ we have

$$\left|\frac{zf'(z)}{f(z)}-1\right| < \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}-\alpha\right\} \Leftrightarrow \quad \operatorname{Re}\left\{1-\frac{zg'(z)}{g(z)}-\alpha\right\} > \left|\frac{zg'(z)}{g(z)}\right| \quad \text{.............(1)}$$

$$\Leftrightarrow 1-\alpha-\operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} > \left|\frac{zg'(z)}{g(z)}\right|$$

$$\Leftrightarrow \operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} < 1-\alpha-\left|\frac{zg'(z)}{g(z)}\right|$$

$$\Leftrightarrow \operatorname{Re}\left\{\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right\} < 1-\alpha-\left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right| \quad \text{.............(2)}$$

The given inequality implies that

$$2\sum_{n=1}^{\infty} n|b_n| < (1-\alpha)(1-\sum_{n=1}^{\infty}|b_n|)$$

$$\Rightarrow 2\frac{\sum_{n=1}^{\infty} n|b_n|}{1-\sum_{n=1}^{\infty}|b_n|} < 1-\alpha$$

$$\Rightarrow 2\left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right| < 1-\alpha$$

$$\Rightarrow \left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right| < 1-\alpha-\left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right|$$

This implies (2) because

$$\operatorname{Re}\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n} \le \left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right|.$$

Thus (1) follows. Hence $f \in S_p(\alpha)$.

Corollary: Let $f(z)= z/(1+\sum_{n=1}^{\infty} b_n z^n) \in A_1$ with b_n's satisfying

$$\sum_{n=1}^{\infty}[2n+1]|b_n| < 1.$$

Then $f(z)$ is in the class S_p.

Section-2

Next we determine a necessary and sufficient condition on a particular form of f in terms of b_n's for f to be in S_p.

Theorem2: $f(z) = z+a_n z^n = z/(1+\sum_{m=1}^{\infty} b_m z^m)$ is in S_p if and only if $|b_{(n-1)m}| \le 1/(2n-1)^m$,

and $b_k = 0$ *for* $k \ne (n-1)m$ for $m \in N$.

Proof: We have

$$f(z)= z/g(z) \text{ where } g(z) = 1+\sum_{m=1}^{\infty} b_m z^m,\ z \in U.$$

For

$$f(z) = z+a_n z^n$$

we have

$$f(z) = \frac{z}{g(z)} = z + a_n z^n,\ z \in U.$$

Hence

$$g(z) = \frac{z}{z+a_n z^n} = \frac{1}{1+a_n z^{n-1}} = 1+\sum_{m=1}^{\infty}\left(-a_n z^{n-1}\right)^m = 1+\sum_{m=1}^{\infty}\left(-a_n\right)^m z^{(n-1)m}$$

$$\Leftrightarrow b_{(n-1)m} = \left(-a_n\right)^m, b_k = 0 \text{ for } k \ne (n-1)m.$$

Now the Theorem2 follows from the above Theorem A

Next we determine a necessary and sufficient condition on a particular form of f in terms of b_n's for f to be in *UCV*.

Theorem3: $f(z) = z+a_n z^n = z/(1+\sum_{m=1}^{\infty} b_m z^m)$ is in *UCV* if and only if

$$|b_{(n-1)m}| \le 1/[n(2n-1)]^m \text{ and } b_k = 0 \text{ for } k \ne (n-1)m \text{ for } m \in N.$$

Proof: Similar to that of Theorem2 via Theorem B

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