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PERIPHERAL TEMPERATURE DISTRIBUTION IN HUMAN SUBJECTS EXPOSED TO WIND FLOW

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Abstract: Finite element method with Crank-Nicolson technique is employed to investigate one dimensional temperature distribution in human skin and subcutaneous tissue (SST) exposed to cold environmental temperature and wind flow. The wind speed is considered in the range from the start of forced convection (≈ 0.2 m/s) and up to 5 m/s. The important parameters like blood mass flow rate, metabolic heat generation rate and thermal conductivity are taken distinct in the layers of SST. The values of these parameters are considered as position dependent in the layers. The loss of heat from the body is considered due to convection, radiation, and insensible perspiration.

Keywords: Bioheat equation, Wind speed, Finite element method, Dermal parts
2000 Mathematical Subject Classification: 92C35, 65L60

Introduction

In a human body the internal body temperature almost remains constant despite the fluctuation of environmental temperatures up to certain limits. The main organ that keeps core temperature constant is dermal part. This organ controls heat flow and moisture flow to and from the surrounding environment [9]. SST plays a key role for heat transfer within human body, and hence there is variation of temperature in SST region in accordance with surrounding temperatures. Skin mainly consists two layers – epidermis and dermis, and the layers of skin also consists sub-layers. The thickness of the layers varies depending on the location. The rate of blood mass flow is strongly related to the concentration of blood vessels in dermal layers. There are no blood vessels in epidermis; the population density of blood vessels in the dermis is very thin near the interface of epidermis and dermis, but it increases gradually and becomes almost uniform in sub-dermal part. This gives us some idea about variation of physical quantities like the rate of blood mass flow, the rate of metabolic heat generation and the thermal conductivity of tissue in this region, in relation to its position. Hence, we confine the variation of parameters in two sub-layers of epidermis like stratum corneum and stratum germinativum; two sub layers of dermis like papillary layer and reticular layer and subcutaneous tissue.

Heat transfer and temperature variation within SST are related to the following factors (i) Heat conduction within tissue (ii) The blood flow rates in arteries and veins (iii) The blood flow perfusion through capillaries in tissues (iv) Metabolic heat generation (v) Exchange between the human skin surface and surrounding (vi) Geometry of the human body and thermal properties of blood and tissues [22]. Thus, the loss of heat from the human body due to cold environment is quantified by convection air flow and radiative heat transfer on the skin surface. The faster the flow of air around the body, the thinner the boundary layer of air on the body's surface, and hence the lower the thermal insulation. The process of convection from heated sources such as human skin is classified as natural convection, and forced convection. The ambient air speed, $v < 0.1$ m/s and ≥ 0.2 m/s are considered respectively for natural convection and forced convection [6]. deDear et al. [6] recommended that the whole body natural convection coefficient for seated and standing thermal manikin are 3.3 and 3.4 W/m^2-K respectively.

Many attempts have been made over the years to empirically define forced convection heat transfer appropriate for the whole human body. The general equation form of the equation describing the dependence of whole body forced convection heat transfer coefficient h_c on air speed v is as follows

$$h_c = Bv^n \text{ (W/m}^2\text{-K)} \qquad (1)$$

with most authors indicating n in the range of 0.5 to 0.6 [6]. deDear et al. [6] measured a value of $B = 10.1$, $n = 0.61$, and $B = 10.4$, $n = 0.58$ for the whole body in static condition both in seated and standing postures, respectively. However, regardless of posture, deDear et al. [6] recommended a general forced convection heat transfer coefficient of the whole body as

$$h_c = 10.3\text{v}^{0.6} \text{ (W/m}^2\text{-K)} \qquad (2)$$

across a range of wind speed from the start of forced convection up to 5 m/s, and the value of 4.5 W/m^2-K for the radiative heat transfer coefficient h_r for the whole body.

Many models have been developed for SST temperature distribution profile at normal and abnormal ambient temperatures in case of still air [8, 11, 18, 19]. The present study considers the effect of air flow with wind speed, v in the range between 0.2 to 5 m/s to predict the temperature variation in dermal layers irrespective of surrounding temperatures.

Bio-heat Equation

Skin heat transfer has been studied extensively due to its importance in thermoregulation. The study involves phenomena that are not found in non-living systems. For example, the study of skin bio-heat transfer includes the phenomena occurred due to blood circulation and metabolism together with the heat conduction process in skin tissue. Pennes' [15] principal contribution was his suggestion that the rate of heat transfer from blood to tissue at any point is proportional to the difference between arterial blood and tissue temperature at that point. He expressed that relationship as follows:

$$h_b = w_b \rho_b c_b (k-1)(T - T_a), \quad 0 \leq k \leq 1 \qquad (4)$$

where h_b is the rate of heat transfer per unit volume of tissue, w_b is the perfusion rate measured in volume of blood per volume per time, ρ_b is the blood density, c_b is the specific heat of blood, k is the equilibrium constant that accounts for incomplete thermal equilibrium between blood and tissue, T and T_a are respectively, the tissue and arterial blood temperature. Pennes set $k = 0$ in his theoretical curve for complete equilibration between capillary blood and tissue.

Following Pennes' suggestion, the thermal energy balance for perfused tissue considers the effects of metabolism and blood perfusion. These two effects were incorporated into the standard thermal diffusion equation, which is in its simplified form as

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T - w_b \rho_b c_b \left(T - T_a\right) + q_m \qquad (5)$$

where ρ and c are density and specific heat of tissue, K is the thermal conductivity of tissue and q_m is the rate of metabolism heat production per unit volume of tissue.

After landmark work of Pennes', several bio-heat transfer equations were suggested [5, 10, 12, 16, 20, 21]. But the researchers preferred using Pennes' bio-heat equation in bio-heat transfer problems due to its computational simplicity and flexibility. So Pennes' bio-heat equation is now-a-days called bio-heat equation.

Perl [16] used equation (5) to illustrate heat distribution and tissue blood flow in body tissues by analyzing existing steady state and transient data obtained by the Gibb's thermoelectric probe method. Chao et al. [3, 4] applied analytic method to solve steady and unsteady models to predict temperature variation in regional dermal layers. Saxena [18] considered the equation (5) to find analytic solution for temperature distribution in the layers of dermal part by taking various values of parameters. Agrawal et al. [1, 2] used it to construct thermal distribution models in dermal layers of elliptical shaped of human limbs involving a uniformly perfused and metastasis of tumor. Gurung et al. [8] applied equation (5) to study unsteady state temperature distribution in dermal parts with quadratic shape function using variational FEM technique. Saxena et al. [19] considered the bio-heat equation (5) to study the heat regulation in human dermal parts with variable heterogeneity. The equation (5) has been used by Pardasani and Shakya [14] to extend the model for infinite element.

Mathematical Model and Method

We re-model the bio-heat equation (5) for one dimensional case by stating sub regional activities distinctly corresponding to the layers of SST region – stratum corneum, stratum germinativum, papillary layer, reticular layer, and subdermal layer as shown in Figure 1, where T_o, T_1, T_2, T_3, T_4, and T_5 are interface nodal temperatures, and l_i (i = 1[1]5) are depth of layer interfaces with thickness x measured along the depth of the tissue from the skin surface. The distinct activities of the layers correspond in the variation of thermal properties in dermal layers even when the same layer, there exist large non-homogeneity and anisotropy due to the presence of blood vessels. Blood mass flow rate and metabolic heat generation rate rely on density of blood vessels at different depth. So it is reasonable to consider $M = w_b \, \rho_b \, c_b$ and q_m negligible in stratum corneum of epidermis, constant throughout in subcutaneous tissue and the linear function of depth in various positions in dermis.

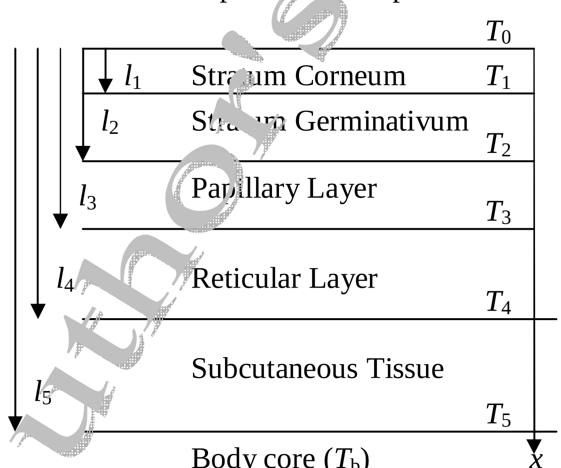

Figure 1. Schematic diagram of dermal layers

Due to the minute thickness of the sub-regions and for the simplicity of calculation, the linear shape functions are considered to approximate the temperature profiles. The linear variation between successive layered has been obtained using Lagrange interpolation method relying on the interface nodal points. Accordingly, we can make the following layer wise assumptions for the physical and physiological parameters to remodel the equation ([illegible])

Stratum Corneum ($0 \le x \le l_1$)

$T^{(1)} = T_0 + \frac{T_1 - T_0}{l_1} x$; $K = K^{(1)}$; $M = M^{(1)} = 0$; $q_m = q_m^{(1)} = 0$;

$T_a = T_a^{(1)} = 0$.

Stratum Germinativum ($l_1 \le x \le l_2$)

$T^{(2)} = \frac{l_2 T_1 - l_1 T_2}{l_2 - l_1} + \frac{T_2 - T_1}{l_2 - l_1} x$; $K = K^{(2)}$; $M = M^{(2)} = 0$;

$q_m = q_m^{(2)} = \left(\frac{x - l_1}{l_4 - l_1}\right) q$; $T_a = T_a^{(1)} = 0$.

Papillary Layer ($l_2 \le x \le l_3$)

$T = T^{(3)} = \frac{l_3 T_2 - l_2 T_3}{l_3 - l_2} + \frac{T_3 - T_2}{l_3 - l_2} x$; $K = K^{(3)}$; $M = M^{(3)} = \left(\frac{x - l_2}{l_4 - l_2}\right) m$;

$q_m = q_m^{(3)} = \left(\frac{x - l_1}{l_4 - l_1}\right) q$; $T_a = T_a^{(3)} = T^{(4)}$.

Reticular Layer ($l_3 \le x \le l_4$)

$T = T^{(4)} = \frac{l_4 T_3 - l_3 T_4}{l_4 - l_3} + \frac{T_4 - T_3}{l_4 - l_3} x$; $K = K^{(4)}$; $M = M^{(4)} = \left(\frac{x - l_2}{l_4 - l_2}\right) m$; $q_m = q_m^{(4)} = \left(\frac{x - l_1}{l_4 - l_1}\right) q$; $T_a = T_a^{(4)} = T^{(5)}$.

Subcutaneous Tissue ($l_4 \le x \le l_5$)

$T = T^{(5)} = \frac{l_5 T_4 - l_4 T_5}{l_5 - l_4} + \frac{T_5 - T_4}{l_5 - l_4} x$; $K = K^{(5)}$; $M = M^{(5)} = m$;

$q_m = q_m^{(4)} = q$ = constatn; $T_a = T_a^{(4)} = T_b$.

All $K^{(i)}$ (i = 1(1)5) have fixed values.

The boundary conditions are taken as:

(i) Heat flux at the skin surface = loss of heat to the environment from skin surface due to combined effect of convection and radiation and sweat evaporation.

(ii) Temperature at the interface of subcutaneous tissue and internal body = deep body core temperature.

In mathematical terms:

$$-K^{(1)} \left. \frac{dT^{(1)}}{dx} \right|_{x=0} = h(T_0 - T_{\text{atm}}) + LE \qquad (6)$$

$$T^{(5)}(x = l_5) = T_5 = T_b = 37^0\text{C} \qquad (7)$$

where $h = h_c + h_r$ is the combined heat transfer coefficient due to convection and radiation; L and E are respectively, the latent heat and sweat evaporation rate, and T_{atm} is the surrounding temperature.

The dermal region of human body is a layered medium. The layers of this region are neither smooth nor distinct from each other. In some cases the geometry of the layer is irregular. Hence, the FEM is a suitable numerical method to find the approximate solution of temperature distribution in dermal region under normal and abnormal atmospheric conditions. The finite element technique allows local autonomy of parameters and is more appropriate than any other numerical method for the estimation of these temperature profiles. We have to change of governing equations into variational form.

The variational integral

$$I = \int F(x, T, T_x)\, dx \qquad (8)$$

in optimum form is equivalent to the Euler-Lagrange differential equation [13, 17].

$$\frac{\partial F}{\partial x} - \frac{d}{dx}\left(\frac{\partial F}{\partial T_x}\right) = 0 \qquad (9)$$

If the finite element equations are derived on the basis of a variational principle, the boundary condition (7) will be automatically incorporated in the formulation; hence only the boundary condition (6) is to be enforced on the solution [17].

Solution

The layer-wise bio-heat equations together with the outer skin surface boundary condition when written for one dimensional unsteady state case and comparing bio-heat equation (5) with Euler-Lagrange equation (9) for one dimensional case, we arrive at the following layer-wise variational integrals [13].

Stratum Corneum

$$I_1 = \frac{1}{2}\int_0^{l_1}\left[K^{(1)}\left(\frac{dT^{(1)}}{dx}\right)^2 + \rho c\frac{\partial\left(T^{(1)}\right)^2}{\partial t}\right]dx + \frac{1}{2}\left[h\left(T_0 - T_{\text{atm}}\right)^2 + 2LET_0\right] \qquad (10a)$$

Stratum Germinativum

$$I_2 = \frac{1}{2}\int_{l_2}^{l_3}\left[K^{(2)}\left(\frac{dT^{(1)}}{dx}\right)^2 - 2q_m^{(2)}T^{(2)} + \rho c\frac{\partial\left(T^{(2)}\right)^2}{\partial t}\right]dx \qquad (10b)$$

Papillary Layer

$$I_3 = \frac{1}{2}\int_{l_2}^{l_3}\left[K^{(3)}\left(\frac{dT^{(3)}}{dx}\right)^2 - M^{(3)}\left(T^{(3)} - T_a^{(3)}\right)^2 - 2q_m^{(3)}T^{(3)} + \rho c\frac{\partial\left(T^{(3)}\right)^2}{\partial t}\right]dx \qquad (10c)$$

Reticular Layer

$$I_4 = \frac{1}{2}\int_{l_3}^{l_4}\left[K^{(4)}\left(\frac{dT^{(4)}}{dx}\right)^2 - M^{(4)}\left(T^{(4)} - T_a^{(4)}\right)^2 - 2q_m^{(4)}T^{(4)} + \rho c\frac{\partial\left(T^{(4)}\right)^2}{\partial t}\right]dx \qquad (10d)$$

Subcutaneous Tissue

$$I_5 = \frac{1}{2}\int_{l_4}^{l_5}\left[K^{(5)}\left(\frac{dT^{(5)}}{dx}\right)^2 - M^{(5)}\left(T^{(5)} - T_a^{(5)}\right)^2 - 2q_m^{(5)}T^{(5)} + \rho c\frac{\partial\left(T^{(5)}\right)^2}{\partial t}\right]dx \quad (10)$$

We assemble above integrals to obtain the unified expression $I = \sum_{i=1}^{5} I_i$

The layer-wise variational integral equations with layer-wise assumptions for physical and physiological parameters give

$$I_1 = A + A_0T_0 + A_{00}T_0^2 + A_{11}T_1^2 + A_{01}T_0T_1 + \frac{\rho c l_1}{6}\frac{d}{dt}\left(T_0^2 + T_1^2 + T_0T_1\right)$$

$$I_2 = B_1T_1 + B_2T_2 + B_{11}T_1^2 + B_{22}T_2^2 + B_{12}T_1T_2 + \frac{\rho c\left(l_2 - l_1\right)}{6}\frac{d}{dt}\left(T_1^2 + T_2^2 + T_1T_2\right)$$

$$I_3 = C_2T_2 + C_3T_3 + C_{22}T_2^2 + C_{33}T_3^2 + C_{44}T_4^2 + C_{23}T_2T_3 + C_{24}T_2T_4 + C_{34}T_3T_4 + \frac{\rho c\left(l_3 - l_2\right)}{6}\frac{d}{dt}\left(T_2^2 + T_3^2 + T_2T_3\right)$$

$$I_4 = D + D_3T_3 + D_4T_4 + D_{33}T_3^2 + D_{44}T_4^2 + D_{34}T_3T_4 + \frac{\rho c\left(l_4 - l_3\right)}{6}\frac{d}{dt}\left(T_3^2 + T_4^2 + T_3T_4\right)$$

$$I_5 = E + E_4T_4 + E_{44}T_4^2 + \frac{\rho c\left(l_5 - l_4\right)}{6}\frac{d}{dt}\left(T_b^2 + T_4^2 + T_bT_4\right)$$

The coefficients pertaining to nodal values are all constants depending upon the value of physical and physiological parameters and are defined in Appendix – A.

To proceed further, we optimize I with respect to nodal values T_i. Thus, we take $\frac{\partial I}{\partial T_i} = 0$ for i = 0, 1, 2, 3, 4. Accordingly, we get the following system of equations

$$P\dot{T} + QT = V \quad (11)$$

where

$$P = \begin{bmatrix} 2a & a & 0 & 0 & 0 \\ a & 2(a+b) & b & 0 & 0 \\ 0 & b & 2(b+c) & c & 0 \\ 0 & 0 & c & 2(c+d) & d \\ 0 & 0 & 0 & d & 2(d+e) \end{bmatrix}$$

$$Q = \begin{bmatrix} 2A_{00} & A_{01} & 0 & 0 & 0 \\ A_{01} & 2(A_{11}+B_{11}) & B_{12} & 0 & 0 \\ 0 & B_{12} & 2(B_{22}+C_{22}) & C_{23} & C_{24} \\ 0 & 0 & C_{23} & 2(C_{33}+D_{33}) & C_{34}+D_{34} \\ 0 & 0 & C_{24} & C_{34}+D_{34} & 2\left(C_{44}+D_{44}+E_{44}\right) \end{bmatrix}$$

$$W = \begin{bmatrix} -A_0 \\ -B_1 \\ -(B_2 + C_2) \\ -(C_3 + D_3) \\ -(D_4 + E_4) \end{bmatrix}; \qquad \dot{T} = \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \\ \frac{dT_3}{dt} \\ \frac{dT_4}{dt} \end{bmatrix}, \quad \text{and} \qquad T = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

with

$$a = \frac{\rho c l_1}{6};\ b = \frac{\rho c (l_2 - l_1)}{6};\ c = \frac{\rho c (l_3 - l_2)}{6};\ d = \frac{\rho c (l_4 - l_3)}{6};\ e = \frac{\rho c (l_5 - l_4)}{6}$$

Now we apply Crank-Nicolson method to solve the system of ordinary differential equations (11). This method gives the solution of the system with regard to time according to the relation

$$\left(P + \frac{\Delta t}{2} Q\right) T^{(i+1)} = \left(P - \frac{\Delta t}{2} Q\right) T^{(i)} + \Delta t W \tag{12}$$

where Δt is time interval.
Here

$$T^{(0)} = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}^{(0)} \text{ at initial time,} \qquad T^{(i)} = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}^{(i)} \text{ and } \qquad T^{(i+1)} = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}^{(i+1)}$$

The initial skin temperature of 32^0C pertains to thermo neutral condition when the human body is exposed to cold environment [7]. Thus, we have

$$T^{(0)} = \begin{bmatrix} 32 \\ 32 \\ 32 \\ 32 \\ 32 \end{bmatrix}$$

Numerical Results and Discussion

The nude human subject is assumed to expose to cold environment with various ambient temperatures. This result constriction of blood vessels causing decrease in blood mass flow rate in blood vessels which in turn decreases the heat carried by blood to the skin surface. This causes increase in metabolic heat generation rate to keep core temperature constant. So, to solve equations (12), we take the following values of physical and physiological parameters [5, 8, 11, 18]

$K^{(1)}$ = 0.209 (W/m ^{0}C)
$K^{(2)}$ = 0.209 (W/m ^{0}C)
$K^{(3)}$ = 0.314 (W/m ^{0}C)
$K^{(4)}$ = 0.314 (W/m ^{0}C)
$K^{(5)}$ = 0.418 (W/m ^{0}C)

M	=	2.093×10^2 (W/m^3-^{0}C)
q	=	3.767×10^3 (W/m^3)
L	=	2.424×10^6 (J/kg)
E	=	0 (kg/m^2-s)
ρ	=	1.05×10^3 (kg/m^3)
c	=	3.474×10^3 (J/kg ^{0}C)
h_r	=	4.5 (W/m^2-^{0}K)
h_c	=	$10.3 \times v^{0.6}$ (W/m^2-^{0}K)

The thickness of various layers of dermal part has been taken as given in Table 1.

Table 1. Thickness of dermal layers considered

Stratum Corneum (m)	Stratum Germinativum (m)	Papillary Layer (m)	Reticular Layer (m)	Subcutaneous Tissue (m)
0.0010	0.0010	0.0015	0.0025	0.0030

It is assumed that the skin surface has been exposed to 50 mins at atmospheric temperatures of 2^{0}C, 0^{0}C and -2^{0}C at different wind speeds of 2 m/s and 4 m/s. The system of equations (12) is solved constructing a computer program for a time step size of 1/60th of a minute. Accordingly, we obtain the values of nodal temperatures at different time interval within 50 mins. The temperature distribution profiles for nodal temperature of nodes lying at the interfaces have been plotted as shown in Figures 2 to 4.

The graphs in figures 2 to 4 represent the variation of nodal tissue temperature and skin surface temperature with the variation of wind speed at environmental temperature. In these figures it can be seen that the variation of tissue temperature occurs more towards skin surface from the body core for various wind speed. This is due to the increase in heat loss from the outer surface of body to the environment for the increase of wind speed. The different properties of physiological parameters of the layers are also accountable for such variation. From this observation we conclude that the wind speed plays an important role in heat loss from the skin surface at lower atmospheric temperatures.

The figures also reveal that tissue temperatures decrease with the decrease in environmental temperature at an equal wind speed. This may be due to more heat loss to the environment at low atmospheric temperatures. In this case, the decrease of tissue temperature is more towards skin surface compared to inner tissue medium. This is because that heat loss occurs through skin surface.

According to present model prediction, the skin surface temperature would reach steady temperature as shown in Table 2 using the parameter values taken.

Table 2. Steady state skin surface temperatures

T_{atm} (^{0}C)	Wind speed (m/s)	Steady temperature (^{0}C)
2	2	7.45
2	4	5.82
0	2	5.75
0	4	4.04
-2	2	4.06
-2	4	2.25

The thermal stress due to cold depends on the skin surface temperatures. Therefore, the model can be employed to predict the ambient temperature due to cold and wind speed to prevent the workers from cold thermal stresses.

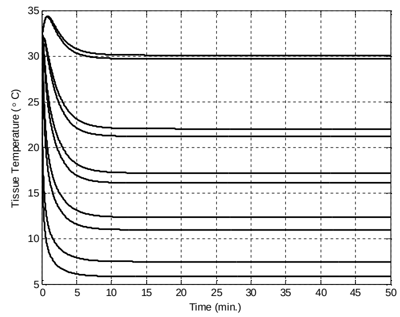

Figure 2. Temperature distribution profiles at the interfaces of layers when $T_{atm} = 2^0C$. The pair of graphs from above to below represents the graph at the interfaces of (i) Reticular and Subcutaneous Tissue (ii) Papillary and Reticular Layers (iii) Stratum Germinativum and Papillary Layer (iv) Stratum Corneum and Stratum Germinativum, and (v) at outer skin surface, respectively. In each pair of graphs, the above one is at wind speed 2 m/s and the below one is at wind speed 4 m/s.

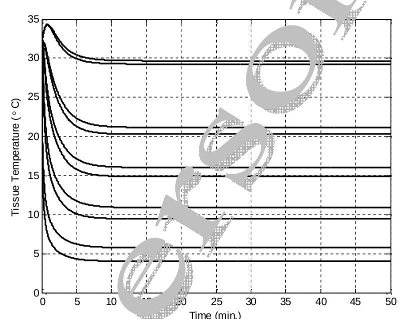

Figure 3. Temperature distribution profiles at the interfaces of layers with $T_{atm} = 0^0C$. The pair of graphs from above to below represents the graph at the interfaces of (i) Reticular and Subcutaneous Tissue (ii) Papillary and Reticular Layers (iii) Stratum Germinativum and Papillary Layer (iv) Stratum Corneum and Stratum Germinativum, and (v) at outer skin surface, respectively. In each pair of graphs, the above one is at wind speed 2 m/s and the below one is at wind speed 4 m/s.

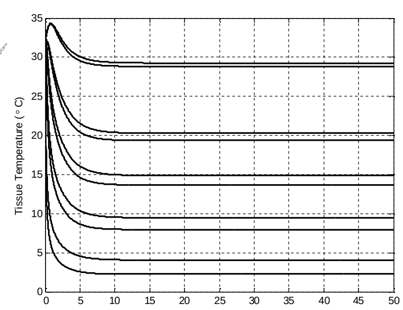

Figure 4. Temperature distribution profiles at the interfaces of layers when $T_{atm} = -2^0C$. The pair of graphs from above to below represents the graph at the interfaces of (i) Reticular and Subcutaneous Tissue (ii) Papillary and Reticular Layers (iii) Stratum Germinativum and Papillary Layer (iv) Stratum Corneum and Stratum Germinativum, and (v) at outer skin surface, respectively. In each pair of graphs, the above one is at wind speed 2 m/s and the below one is at wind speed 4 m/s.

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Appendix A

$$A = \frac{hT_a^2}{2}; \quad A_0 = LE - hT_a; \quad A_{00} = \frac{1}{2}\left(\frac{K^{(1)}}{l_1} + h\right); \quad A_{11} = \frac{1}{2}\frac{K^{(1)}}{l_1}; \quad A_{01} = -\frac{K^{(1)}}{l_1};$$

$$B_1 = -\frac{(l_2 - l_1)^2}{6(l_4 - l_1)}q; \; B_2 = 2B_1; \; B_{11} = \frac{K^{(2)}}{2(l_2 - l_1)}; \; B_{22} = B_{11}; \; B_{12} = -2B_{11};$$

$$C_2 = \alpha_{31}; \; C_3 = \alpha_{32}; \; C_{22} = \alpha + b_{30}; \; C_{33} = \alpha + b_{31}; \; C_{44} = b_{32}; \; C_{23} = -2\alpha + b_{33}; \; C_{24} = b_{34};$$

$$C_{34} = b_{35};$$

where,

$$\alpha_{30} = -\frac{(l_3 + l_2 - 2l_1)}{2(l_4 - l_1)}q;$$

$$\beta_{30} = -\frac{[2(l_3^2 + l_2 l_3 + l_2^2) - 3l_1(l_3 + l_2)]}{6(l_4 - l_1)}q;$$

$$\alpha = \frac{K^{(3)}}{2(l_3 - l_2)};$$

$$\alpha_{31} = \alpha_{30}l_3 - \beta_{30};$$

$$\alpha_{32} = -\alpha_{30}l_2 + \beta_{30};$$

$$b_{30} = f_{31} + g_{31} + h_{31};$$

$$b_{31} = c_{31} + d_{31} + e_{31} + f_{32} + g_{32} + h_{32} + i_{31} + j_{31} + k_{31} + l_{31};$$

$$b_{32} = c_{32} + d_{32} + e_{32};$$

$$b_{33} = f_{33} + g_{33} + h_{33} + i_{32} + j_{32} + k_{32} + l_{32};$$

$$b_{34} = i_{33} + j_{33} + k_{33} + l_{33};$$

$$b_{35} = c_{33} + d_{33} + e_{33} + i_{34} + j_{34} + k_{34} + l_{34};$$

$$c_{30}=\frac{(l_3-l_2)^2}{4(l_4-l_2)(l_4-l_3)^2}m;\qquad c_{31}=l_4^2c_{30};\qquad c_{32}=l_3^2c_{30};\qquad c_{33}=-2l_3l_4c_{30};$$

$$d_{30}=\frac{\left[3\left(l_3^4-l_2^4\right)-4l_2\left(l_3^3-l_2^3\right)\right]}{24(l_4-l_2)(l_4-l_3)^2}m;\qquad d_{31}=d_{30};\qquad d_{32}=d_{30};\qquad d_{33}=-2d_{30};$$

$$e_{30}=\frac{\left[3\left(l_3^3-l_2^3\right)-3l_2\left(l_3^2-l_2^2\right)\right]}{6(l_4-l_2)(l_4-l_3)^2}m;\ e_{31}=-l_4e_{30};\ e_{32}=-l_3e_{30};\ e_{33}=(l_3+l_4)e_{30};\ f_{30}=\frac{m}{4(l_4-l_2)};$$

$$f_{31}=l_3^2f_{30};\ f_{32}=l_2^2f_{30};\ f_{33}=-2l_2l_3f_{30};\ g_{30}=\frac{\left[3\left(l_3^2+l_2^3\right)(l_2+l_3)-4l_2\left(l_2^2+l_2l_3+l_3^2\right)\right]}{24(l_3-l_2)(l_4-l_2)}m;$$

$$g_{31}=g_{30};\qquad g_{32}=g_{30};\qquad g_{33}=-2g_{30};\qquad h_{30}=\frac{\left[2\left(l_3^3-l_2^3\right)-3l_2\left(l_3^2-l_2^2\right)\right]}{6(l_4-l_2)(l_4-l_3)^2}m;\qquad h_{31}=-l_3h_{30};$$

$$h_{32}=-l_2h_{30};\ h_{33}=(l_2+l_3)h_{30};$$

$$i_{30}=-\frac{(l_3-l_2)}{2(l_4-l_3)(l_4-l_2)}m;\qquad i_{31}=-l_2l_4i_{30};\qquad i_{32}=l_3l_4i_{30};\qquad i_{33}=-l_3^2i_{30};\qquad i_{34}=l_2l_3i_{30};$$

$$j_{30}=-\frac{\left[2\left(l_3^3-l_2^3\right)-3l_2\left(l_3^2-l_2^2\right)\right]}{6(l_3-l_2)(l_4-l_2)(l_4-l_3)}m;\ j_{31}=l_4j_{30};\ j_{32}=-l_4j_{30};\ j_{33}=l_3j_{30};\ j_{34}=-l_3j_{30};$$

$$k_{30}=j_{30};\qquad k_{31}=l_2k_{30};\qquad k_{32}=-l_3k_{30};\qquad k_{33}=l_3k_{30};\qquad k_{34}=-l_2k_{30};$$

$$l_{30}=-\frac{\left[3\left(l_3^4-l_2^4\right)-4l_2\left(l_3^3-l_2^3\right)\right]}{12(l_3-l_2)(l_4-l_2)(l_4-l_3)}m;\ l_{31}=-l_{30};\ l_{32}=l_{30};\ l_{33}=-l_{30};\ l_{34}=l_{30};$$

$$D=b_{42}T_b^2;\quad D_3=\alpha_{41}+b_{44}T_b;\quad D_4=\alpha_{42}+b_{45}T_b;\quad D_{33}=\beta+b_{40};\quad D_{44}=\beta+b_{41};$$
$$D_{34}=-2\beta+b_{43};$$

where,

$$\alpha_{40}=-\frac{(l_3+l_4-2l_1)}{2(l_4-l_1)}q;\ \beta_{40}=-\frac{\left[2\left(l_4^2+l_3l_4+l_3^2\right)-3l_1(l_3+l_4)\right]}{6(l_4-l_1)}q;$$

$$\beta=\frac{K^{(4)}}{2(l_4-l_3)};\qquad \alpha_{41}=\alpha_{40}l_4-\beta_{40};\qquad \alpha_{42}=-\alpha_{40}l_3+\beta_{40};\qquad b_{40}=f_{31}+g_{31}+h_{31};$$

$$b_{41}=c_{41}+d_{41}+e_{41}+f_{42}+g_{42}+h_{42}+i_{41}+j_{41}+k_{41}+l_{41};\qquad b_{42}=c_{42}+d_{42}+e_{42};$$

$$b_{43}=f_{43}+g_{43}+h_{43}+i_{42}+j_{42}+k_{42}+l_{42};\qquad b_{44}=i_{43}+j_{43}+k_{43}+l_{43};$$

$$b_{45}=c_{43}+d_{43}+e_{43}+i_{44}+j_{44}+k_{44}+l_{44};\qquad c_{40}=\frac{(l_4-l_3)(l_4+l_3-2l_2)}{4(l_4-l_2)(l_5-l_4)^2}m;\qquad c_{41}=l_5^2c_{40};$$

$$c_{42}=l_4^2c_{40};\qquad c_{43}=-2l_4l_5c_{40};\qquad d_{40}=\frac{\left[3\left(l_4^4-l_3^4\right)-4l_2\left(l_4^3-l_3^3\right)\right]}{24(l_4-l_2)(l_5-l_4)^2}m;\qquad d_{41}=d_{40};\qquad d_{42}=d_{40};$$

$$d_{43}=-2d_{40};\quad e_{40}=\frac{\left[3\left(l_4^3-l_3^3\right)-3l_2\left(l_4^2-l_3^2\right)\right]}{6(l_4-l_2)(l_5-l_4)^2}m;\quad e_{41}=-l_5e_{40};\quad e_{42}=-l_4e_{40};\quad e_{43}=(l_3+l_4)e_{40};$$

$$f_{40} = \frac{(l_4 + l_3 - 2l_2)}{4(l_4 - l_2)(l_4 - l_3)}m; \quad f_{41} = l_4^2 f_{40}; \quad f_{42} = l_3^2 f_{40}; \quad f_{43} = -2l_3 l_4 f_{40};$$

$$g_{40} = \frac{\left[3\left(l_4^2 + l_3^3\right)(l_4 + l_3) - 4l_2\left(l_3^2 + l_3 l_4 + l_4^2\right)\right]}{24(l_4 - l_3)(l_4 - l_2)}m; \quad g_{41} = g_{40}; \quad g_{42} = g_{40}; \quad g_{43} = -2g_{40};$$

$$h_{40} = \frac{\left[2\left(l_4^3 - l_3^3\right) - 3l_2\left(l_4^2 - l_3^2\right)\right]}{6(l_4 - l_2)(l_4 - l_3)^2}m; \; h_{41} = -l_4 h_{40}; \; h_{42} = -l_3 h_{40}; \; h_{43} = (l_3 + l_4)h_{40};$$

$$i_{40} = -\frac{(l_3 + l_4 - 2l_2)}{2(l_5 - l_4)(l_4 - l_2)}m; \quad i_{41} = -l_3 l_5 i_{40}; \quad i_{42} = l_4 l_5 i_{40}; \quad i_{43} = -l_4^2 i_{40}; \quad i_{44} = l_3 l_4 i_{40};$$

$$j_{40} = -\frac{\left[2\left(l_4^3 - l_3^3\right) - 3l_2\left(l_4^2 - l_3^2\right)\right]}{6(l_5 - l_4)(l_4 - l_3)(l_4 - l_2)}m; \; j_{41} = l_5 j_{40}; \; j_{42} = -l_5 j_{40}; \; j_{43} = l_4 j_{40}; \; j_{44} = -l_4 j_{40};$$

$$k_{40} = j_{40}; \quad k_{41} = l_3 k_{40}; \quad k_{42} = -l_4 k_{40}; \quad k_{43} = l_4 k_{40}; \quad k_{44} = -l_3 k_{40};$$

$$l_{40} = -\frac{\left[3\left(l_4^4 - l_3^4\right) - 4l_2\left(l_4^3 - l_3^3\right)\right]}{12(l_5 - l_4)(l_4 - l_3)(l_4 - l_2)}m; \; l_{41} = -l_{40}; \; l_{42} = l_{40}; \; l_{43} = -l_{40}; \; l_{44} = l_{40};$$

$E = b_{50} + \gamma T_b^2 + (a_{52} + b_{52})T_b + b_{54} T_b^2;$

$E_4 = (-2\gamma + b_{55})T_b + a_{51} + b_{51};$

$E_{44} = \gamma + b_{53}$

where,

$$a_{51} = \frac{(l_4 - l_5)}{2}q; \quad a_{52} = a_{51}; \quad \gamma = -\frac{K^{(\ldots)}}{2(l_5 - l_4)}; \quad c_{50} = \frac{T_b^2(l_5 - l_4)}{3}m; \quad d_{50} = \frac{m}{2(l_5 - l_4)};$$

$$e_{50} = \frac{\left(l_5^3 - l_4^3\right)m}{6(l_5 - l_4)^2}; \; f_{50} = \frac{(l_5 + l_4)m}{2(l_5 - l_4)}; \; g_{50} = -mT_b; \; h_{50} = -\frac{T_b(l_5 + l_4)m}{2}$$

$d_{51} = l_5^2 d_{50}; \; d_{52} = l_4^2 d_{50}; \; d_{53} = -2l_4 l_5 d_{50}; \; e_{51} = e_{50}; \; e_{52} = e_{50}; \; e_{53} = -2e_{50};$

$f_{51} = -l_5 f_{50}; \; f_{52} = -l_4 f_{50}; \; f_{53} = (l_4 + l_5) f_{50}; \; g_{51} = l_5 g_{50}; \; g_{52} = -l_4 g_{50};$

$h_{51} = -h_{50}; \; h_{52} = h_{50}; \; b_{50} = c_{50}; \; b_{51} = g_{51} + h_{51}; \; b_{52} = g_{52} + h_{52}; \; b_{53} = d_{51} + e_{51} + f_{51};$

$b_{54} = d_{52} + e_{52} + f_{52}; \; b_{55} = d_{53} + e_{53} + f_{53};$