

**MHD EFFECTS ON FLOW PAST AN EXPONENTIALLY  
ACCELERATED VERTICAL PLATE WITH VARIABLE  
TEMPERATURE**

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**Abstract**

MHD flow past an exponentially accelerated infinite vertical plate with variable temperature is studied here. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and temperature are studied for different physical parameters like magnetic field parameter, thermal Grashof number, time and an accelerating parameter  $a$ . It is observed that for an increase in magnetic field parameter  $M$ , there is a fall in the velocity.

**Key words:** exponential, accelerated vertical plate, magnetic field

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**Nomenclature**

$A$  - constant

$a$  - accelerating parameter

$a$  - dimensionless accelerating parameter

$B_0$  - magnetic field strength

$C_p$  - specific heat at constant pressure

$g$  - acceleration due to gravity

$Gr$  - thermal Grashof number

$k$  - thermal conductivity of the fluid

$k^*$  - mean absorption coefficient

$M$  - magnetic field parameter.

$Pr$  - Prandtl number

$p$  - pressure

$T$  - temperature of the fluid near the plate

$T_w$  - temperature of the plate

$T_{\infty}$  - temperature of the fluid far away from the plate  
 $t$  - time  
 $t^*$  - dimensionless time  
 $u$  - velocity of the fluid in the x-direction  
 $u_0$  - velocity of the plate  
 $u^*$  - dimensionless velocity  
 $y$  - coordinate axis normal to the plate  
 $y^*$  - dimensionless coordinate axis normal to the plate

### Greek symbols

$\alpha$  - thermal diffusivity  
 $\beta$  - volumetric coefficient of thermal expansion  
 $\mu$  - coefficient of viscosity  
 $\nu$  - kinematic viscosity  
 $\rho$  - density  
 $\sigma$  - stefan-Boltzmann constant  
 $\tau$  - dimensionless skin-friction  
 $\theta$  - dimensionless temperature  
 $\eta$  - similarity parameter  
erfc - complementary error function

### Introduction

Magneto convection plays an important in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals.

Sakiadis [2,3] studied the growth of the two dimensional velocity boundary layer over a continuously moving horizontal plate emerging from a wide slot at uniform velocity. Soundalgekar [5] was the first to present an exact solution for the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through Gr. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [4]. The Skin-friction for accelerated vertical plate has been studied analytically by Hossian and Shayo [1].

The object of the present paper is to study the MHD effects on flow past an exponentially accelerated infinite vertical plate with variable temperature. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

### Mathematical Formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature in the presence of magnetic field is considered. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time  $t' \leq 0$ , the plate and fluid are at the same temperature  $T_\infty$ . At time  $t' > 0$ , the plate is exponentially accelerated with a velocity  $u = u_0 \exp(at')$  in its own plane and the plate temperature is made to raise linearly with time. A transverse magnetic field of uniform strength  $B_0$  is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. Then under usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g \beta (T - T_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \quad (2)$$

With the following initial and boundary conditions

$$\begin{aligned} &u' = 0, & T = T_\infty & \text{for all } y', t' \leq 0 \\ t' > 0: & u' = u_0 \exp(at), & T = T_\infty + (T_w - T_\infty) A t' & \text{at } y' = 0 \\ & u' \rightarrow 0, & T \rightarrow T_\infty & \text{as } y' \rightarrow \infty \end{aligned} \quad (3)$$

Where  $A = \frac{u_0^2}{\nu}$

On introducing the following non-dimensional quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad a = \frac{a' \nu}{u_0^2} \quad (4)$$

$$Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$$

in equations (1) to (3), leads to

$$\frac{\partial u}{\partial t} = Gr \theta + \frac{\partial^2 u}{\partial y^2} - M u \quad (5)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (6)$$

The initial and boundary conditions in a non-dimensional form are

$$\begin{aligned} &u = 0, & \theta = 0 & \text{for all } y, t \leq 0 \\ t > 0: & u = \exp(at), & \theta = t & \text{at } y = 0 \\ & u \rightarrow 0, & \theta \rightarrow 0 & \text{for all } y \rightarrow \infty \end{aligned} \quad (7)$$

The dimensionless governing equations (8) and (9), subject to the boundary conditions (10), are solved by the usual Laplace-transform technique and the solutions are derived as follows :

$$\theta = t \left[ (1 + 2\eta^2 \text{Pr}) \operatorname{erfc}(\eta\sqrt{\text{Pr}}) - \frac{2}{\sqrt{\pi}} \eta\sqrt{\text{Pr}} e^{-\eta^2 \text{Pr}} \right] \quad (8)$$

$$\begin{aligned} u = & \frac{e^{at}}{2} \left[ e^{-2\eta\sqrt{(a+M)t}} \operatorname{erfc}(\eta - \sqrt{(a+M)t}) + e^{2\eta\sqrt{(a+M)t}} \operatorname{erfc}(\eta + \sqrt{(a+M)t}) \right] \\ & + \frac{Gr(1+bt)}{2(1-\text{Pr})b^2} \left[ e^{-2\eta\sqrt{Mt}} \operatorname{erfc}(\eta - \sqrt{Mt}) + e^{2\eta\sqrt{Mt}} \operatorname{erfc}(\eta + \sqrt{Mt}) \right] \\ & - \frac{Gr\eta\sqrt{t}}{2(1-\text{Pr})b\sqrt{M}} \left[ e^{-2\eta\sqrt{Mt}} \operatorname{erfc}(\eta - \sqrt{Mt}) - e^{2\eta\sqrt{Mt}} \operatorname{erfc}(\eta + \sqrt{Mt}) \right] \\ & - \frac{Gr e^{bt}}{2(1-\text{Pr})b^2} \left[ e^{-2\eta\sqrt{(b+M)t}} \operatorname{erfc}(\eta - \sqrt{(b+M)t}) + e^{2\eta\sqrt{(b+M)t}} \operatorname{erfc}(\eta + \sqrt{(b+M)t}) \right] \\ & - \frac{Gr}{(1-\text{Pr})b^2} \operatorname{erfc}(\eta\sqrt{\text{Pr}}) - \frac{Gr t}{(1-\text{Pr})b} \left[ (1 + 2\eta^2 \text{Pr}) \operatorname{erfc}(\eta\sqrt{\text{Pr}}) - \frac{2}{\sqrt{\pi}} \eta\sqrt{\text{Pr}} e^{-\eta^2 \text{Pr}} \right] \\ & + \frac{Gr e^{bt}}{2(1-\text{Pr})b^2} \left[ e^{-2\eta\sqrt{b\text{Pr}t}} \operatorname{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{bt}) + e^{2\eta\sqrt{b\text{Pr}t}} \operatorname{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{bt}) \right] \end{aligned} \quad (9)$$

where  $\eta = \frac{y}{2\sqrt{t}}$  and  $b = \frac{M}{1-\text{Pr}}$

### Results and Discussion

For physical understanding of the problem numerical computations are carried out for different parameters  $M$ ,  $a$ ,  $Gr$  and  $t$  upon the nature of the flow and transport. The numerical values of the velocity and temperature are computed for different physical parameters like  $M$ ,  $a$ , Prandtl number, thermal Grashof number and time.

The velocity profiles for different values of the magnetic field parameter are shown in Fig.1. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different values of ( $a = 0.2, 0.5, 0.8$ ) and time ( $t = 0.2, 0.4, 0.6$ ) are shown in the Fig.2. It is observed that the velocity increases with increasing values of  $a$  or time.

The velocity profiles for different thermal Grashof number ( $Gr = 2, 10$ ) are shown in the Fig.3. It is observed that velocity increases with increasing values of  $Gr$ .

The temperature profiles are calculated for different values of the time ( $t = 0.2, 0.4, 0.6$ ) are shown in Fig.4. It is observed that temperature increases with increasing values of time.

From the velocity field, we now study the skin. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0} \quad (10)$$

Hence, from the equations (9) and (10), the wall shear stress in the presence of magnetic field is as follows.

$$\begin{aligned} \tau = & \frac{e^{at}}{\sqrt{\pi t}} \left[ \sqrt{\pi(a+M)t} \operatorname{erf}(\sqrt{(a+M)t}) + 1 \right] \\ & + \frac{Gr(1+bt)}{\sqrt{\pi t(1-Pr)b^2}} \left[ \sqrt{\pi Mt} \operatorname{erf}(\sqrt{Mt}) + 1 \right] + \frac{Gr}{2(1-Pr)b\sqrt{M}} \operatorname{erf}(\sqrt{Mt}) \\ & - \frac{Gr e^{bt}}{\sqrt{\pi t(1-Pr)b^2}} \left[ \sqrt{\pi(b+M)t} \operatorname{erf}(\sqrt{(b+M)t}) + 1 \right] - \frac{Gr\sqrt{Pr}}{\sqrt{\pi t(1-Pr)b^2}} \\ & - \frac{2Gr t\sqrt{Pr}}{\sqrt{\pi t(1-Pr)b}} + \frac{Gr e^{ct}}{\sqrt{\pi t(1-Pr)b^2}} \left[ \sqrt{\pi b Pr t} \operatorname{erf}(\sqrt{bt}) + \sqrt{Pr} \right] \end{aligned}$$

The numerical values of  $\tau$  are presented in Table 1. It is observed that from this Table, skin-friction increases with increasing values of the magnetic field parameter, accelerating parameter and time. It is also observed that the skin-friction decreases with increasing values of the Grashof number.

**Table 1:** Values of the non-dimensional skin-friction

M	Gr	a	t	Pr=0.71	Pr=7.0
2	5	2	0.2	3.9221	2.8795
5	5	2	0.2	5.1432	4.7579
10	5	2	0.2	6.6444	6.4633
2	5	0	0.2	1.8217	0.7791
2	5	5	0.2	9.6140	8.5714
2	5	2	0.4	5.1887	3.7964
2	10	2	0.2	3.5929	1.5077

## Conclusions

The theoretical solution of flow past an exponentially accelerated infinite vertical plate with variable temperature in the presence of magnetic field is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, thermal Grashof number,  $a$  and  $t$  are studied graphically. It is observed that the velocity decreases with increasing values of magnetic field parameter but increases with increasing values of  $Gr$ ,  $a$  and  $t$ .

## References

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## Figure Captions:

- Figure 1. Velocity profiles for different  $M$   
Figure 2. Velocity profiles for different  $a$  and  $t$   
Figure 3. Velocity profiles for different  $Gr$   
Figure 4. Temperature profiles for different  $t$

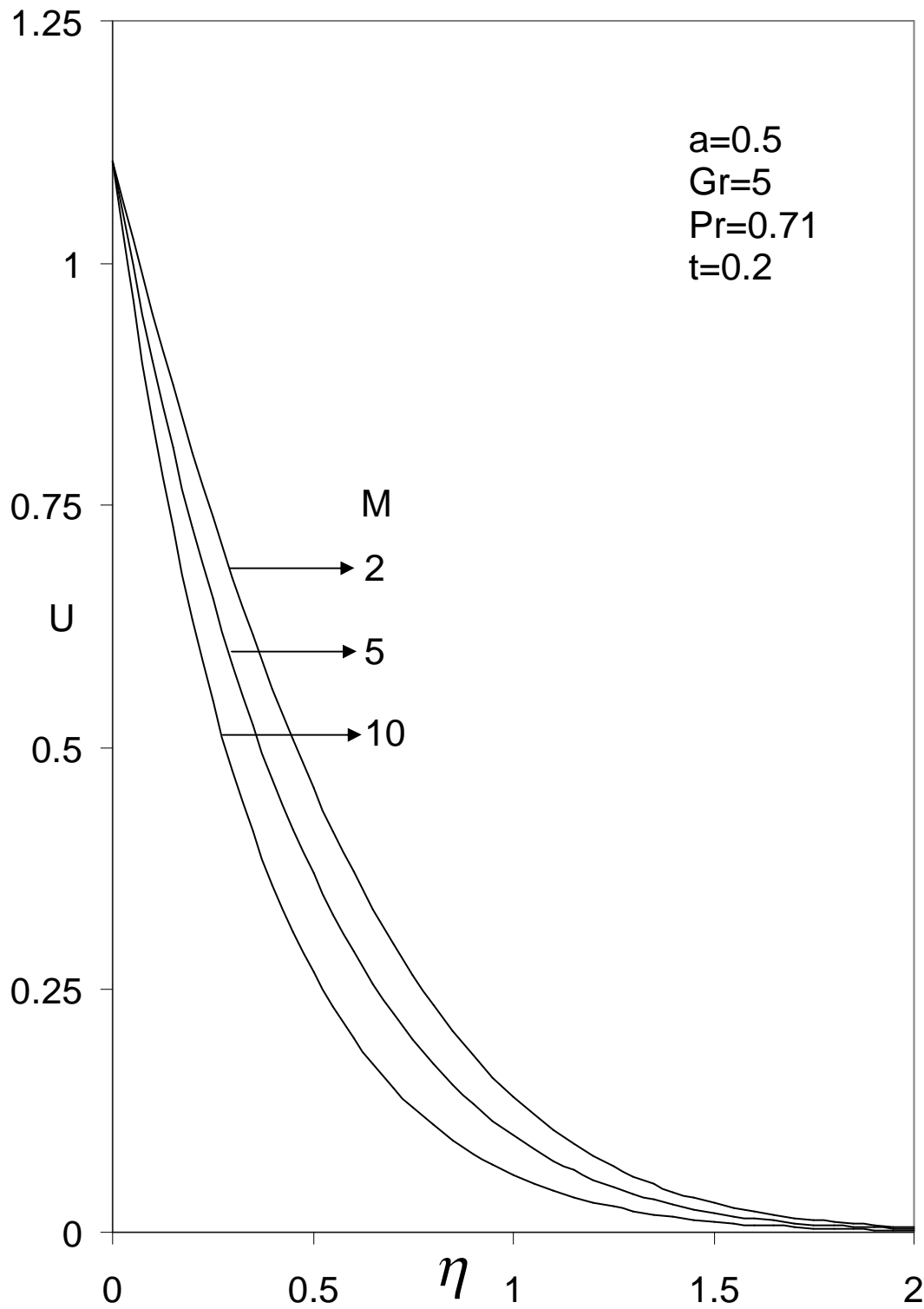


Figure 1: Velocity profiles for different M



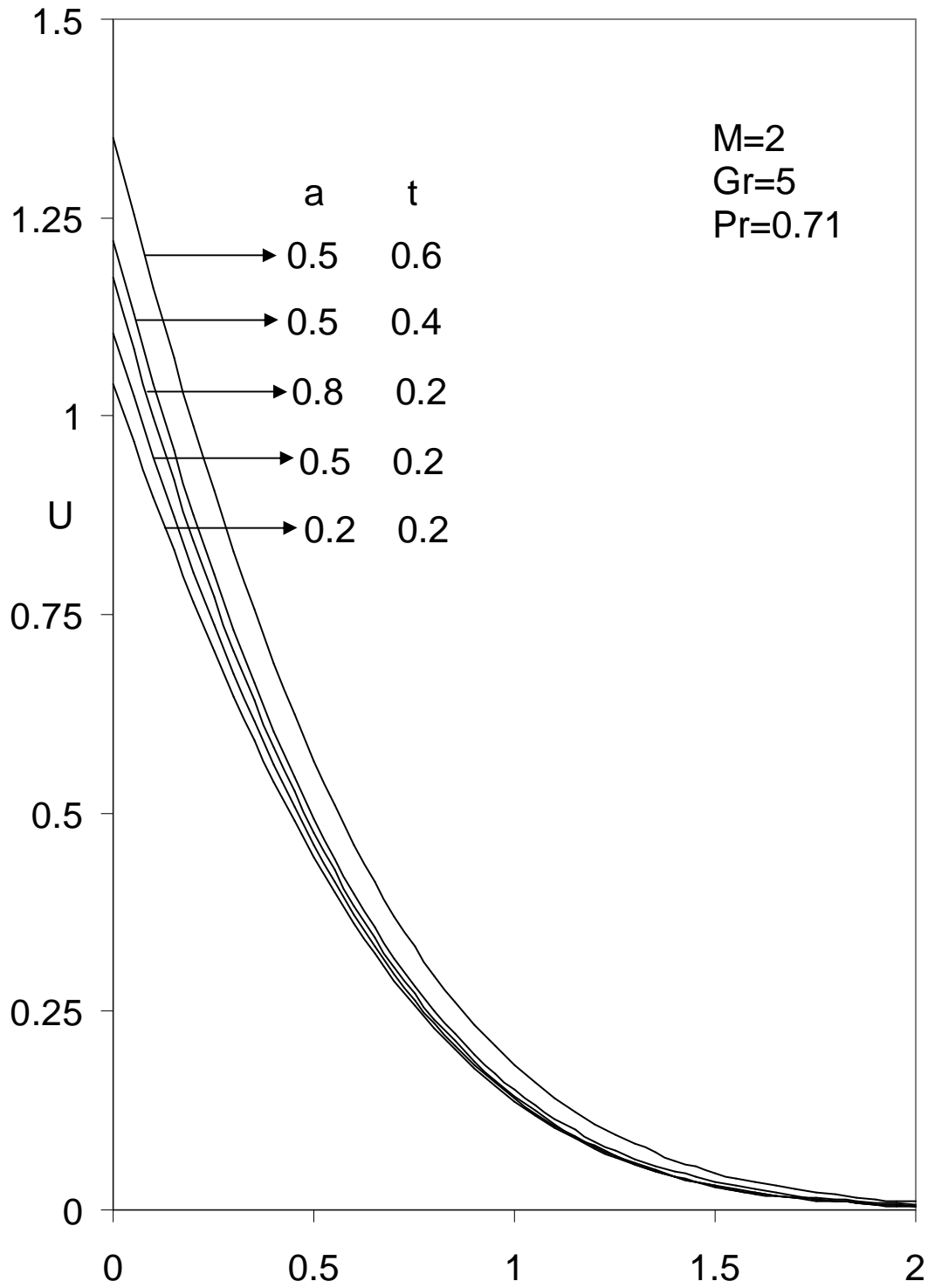


Figure 2: Velocity profiles for different a and t



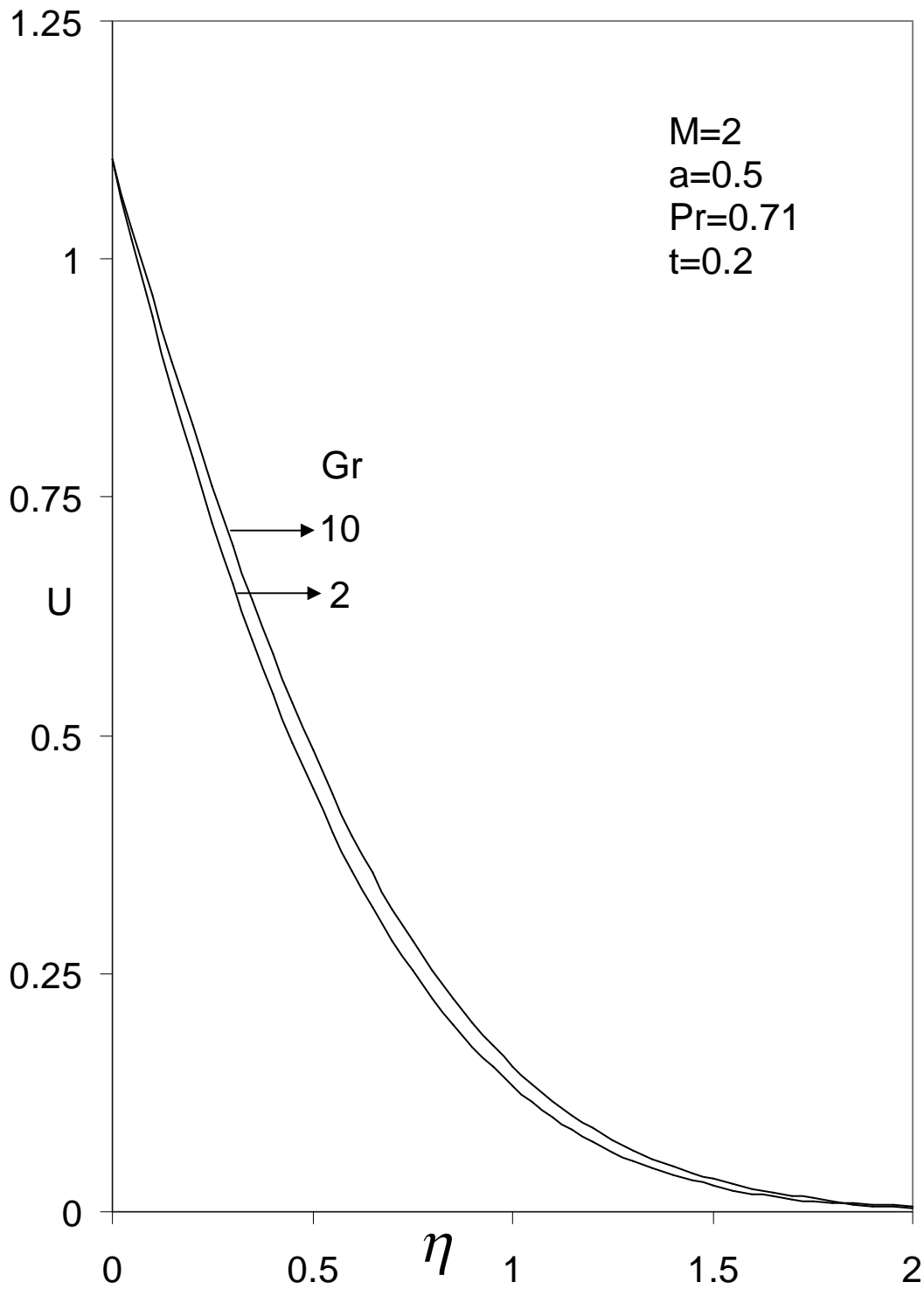


Figure 3: Velocity profiles for different Gr

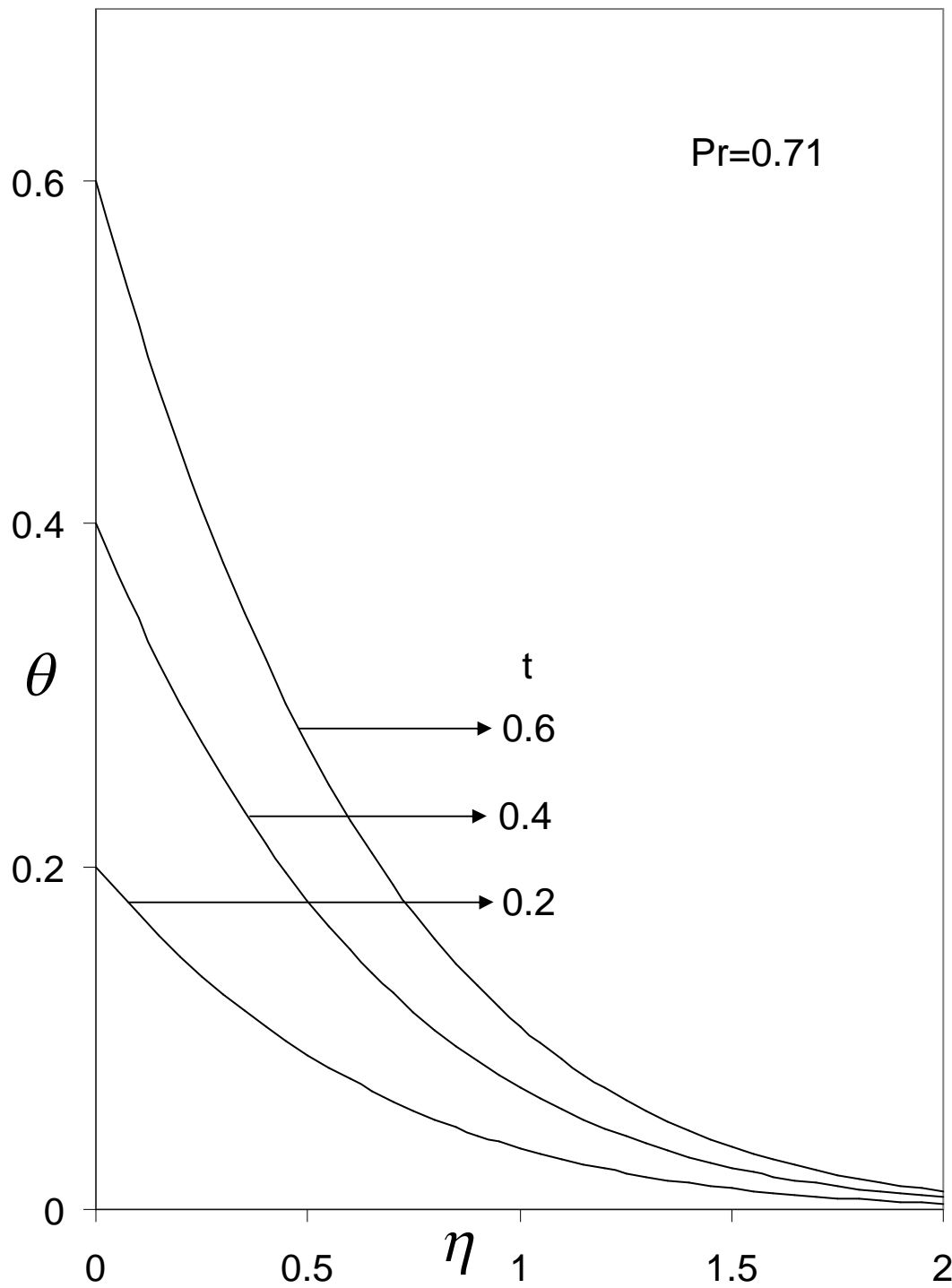


Figure 4: Temperature profiles for different t