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INVERSE COEFFICIENT CRITERIA FOR k-ST AND Re{A FUNCTIONAL}

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Abstract:
A normalized function f analytic in the open unit disc around the origin and non vanishing outside the origin can be expressed in the form z/g(z) where g(z) has Taylor coefficients b_n's. Coefficient conditions in terms of b_n's are derived for functions in the class k- ST of univalent analytic functions and Re {a functional} of analytic functions.

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Introduction:

Let A_1 be the class of functions f analytic in $U = \{ z\in \boldsymbol{C}: |z|<1\}$, and normalized by $f(0)=0$, $f'(0)=1$ where $\boldsymbol{C}$ is the set of complex numbers. An f in A_1 with $f(z)\neq 0$ in the punctured disc $U\backslash\{0\}$, may be expressed as $f(z) = \psi(g) = z / g(z)$ in U, where $g(z) = 1 + \sum_{n=1}^{\infty} b_n z^n$ in U. We call b_n's , the inverse coefficients of f.

Mitrinovic [2],Prawitz[3], Reade et.al [4] ,Silverman and Silvia[5] and Srinivas[6,7,8] studied these coefficients b_n's.

Mitrinovic [2] obtained estimates for the radius of univalence of certain rational functions. In particular, he found sufficient conditions for functions of the form

$$\frac{z}{1+b_1 z + b_2 z^2 + \ldots + b_n z^n},$$

$b_n \neq 0$, to be univalent in the unit disk U.

Prawitz [3] determined the following necessary condition in terms of b_n's of $g(z)$ when $f=\psi(g)$ is in $\boldsymbol{S}$, the subclass of A_1 of functions univalent in U:

$$\sum_{n=2}^{\infty}(n-1)|b_n|^2 \le 1. \qquad \ldots\ldots\ldots.. \quad (1)$$

Silverman and Silvia [5] found necessary conditions in terms of b_n's for $\psi(g)$ to be starlike of order α with negative Taylor coefficients . They found that for such $\psi(g)$

$$|b_n| \le \frac{1-\alpha}{n+1-\alpha}, \quad n = 0,1,2,...$$

Reade et al [4] found that , if

$$\sum_{n=2}^{\infty}(n+1-\alpha) \le \begin{cases}(1-\alpha)-(1-\alpha)|b_1|, 0 \le \alpha \le 1/2 \\ (1-\alpha)-\alpha|b_1|, \quad 1/2 \le \alpha \le 1.\end{cases} \qquad \ldots\ldots\ldots\ldots(2)$$

then $\psi(g) \in S^*(\alpha) \equiv \left\{ f \in A_1 : \mathrm{Re}\frac{zf'(z)}{f(z)} > \alpha, z \in U \right\}.$

Kanas and Wisniowska[1] introduced the class of k-starlike functions , for $0 \le k < \infty$, defined as

$$k\text{-}ST \equiv \left\{ f \in S : \mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > k\left|\frac{zf'(z)}{f(z)}-1\right|, z \in U \right\}, \text{ where } \boldsymbol{S} \equiv S.$$

Let $B(\alpha) \equiv \left\{ f \in A_1 : \mathrm{Re}\frac{f(z)}{z} > \alpha, z \in U \right\}$, *for* $0 \le \alpha \le 1.$ Srinivas [6] found sufficient condition on $B(\alpha)$ as

i) *if* $\sum_{n=1}^{\infty}|b_n| \le 1$, *then* $f \in B(\alpha)$ *for* $0 \le \alpha \le 1/2$

ii) if $\sum_{n=1}^{\infty}|b_n| \le \frac{1-\alpha}{\alpha}$, *then* $f \in B(\alpha)$ *for* $1/2 \le \alpha \le 1$. $\qquad \ldots\ldots\ldots\ldots(3)$

In this paper some necessary conditions on b_n*'s* for functions in the class *k-ST* and *Re{a particular functional}* on A_1 are derived.

Section1

First we derive a sufficient condition for the class *k-ST*:

Theorem1. Let $f(z) = z/(1+\sum_{n=1}^{\infty}b_n z^n) \in A_1$ with b_n's satisfying

$$\sum_{n=1}^{\infty}[n(1+k)+1]|b_n| < 1 \qquad \ldots\ldots\ldots\ldots(4)$$

Then $f(z)$ is in the class *k-ST*.

Proof: For $f(z)=z/g(z)$ where $g(z) = 1+\sum_{n=1}^{\infty}b_n z^n, z \in U,$ we have

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > k\left|\frac{zf'(z)}{f(z)}-1\right| \qquad \ldots\ldots\ldots(5)$$

$$\Leftrightarrow \quad \mathrm{Re}\left\{1-\frac{zg'(z)}{g(z)}\right\} > k\left|\frac{zg'(z)}{g(z)}\right|$$

$$\Leftrightarrow\ 1-\operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} > k\left|\frac{zg'(z)}{g(z)}\right|$$

$$\Leftrightarrow\ \operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} < 1-k\left|\frac{zg'(z)}{g(z)}\right| \qquad \text{……….(6)}$$

Consider $\left|\frac{zg'(z)}{g(z)}\right| < 1-k\left|\frac{zg'(z)}{g(z)}\right|$ …….(7).

The given inequality (4) implies that $\sum_{n=1}^{\infty}\left[n(1+k)\right]|b_n| < 1-\sum_{n=1}^{\infty}|b_n|$

$$\Leftrightarrow \frac{\sum_{n=1}^{\infty} n|b_n|}{1-\sum_{n=1}^{\infty}|b_n|} < \frac{1}{1+k}$$

$$\Rightarrow \left|\frac{zg'(z)}{g(z)}\right| = \left|\frac{\sum_{n=1}^{\infty} nb_n z^n}{1+\sum_{n=1}^{\infty} b_n z^n}\right| \leq \frac{\sum_{n=1}^{\infty} n|b_n|}{1-\sum_{n=1}^{\infty}|b_n|} < \frac{1}{1+k}$$

$$\Rightarrow \left|\frac{zg'(z)}{g(z)}\right| < \frac{1}{1+k}$$

This gives $(7)\Rightarrow(6)\Rightarrow(5)$ because $\operatorname{Re}\xi \leq |\xi|, \xi$ *complex*.

Hence $f \in k$- *ST*.

Section2

Next we derive a sufficient criterion on b_n's for Re{a functional} on A_1 to be positive on *U*.

Theorem2. If $\psi(g) = \dfrac{z}{1+\sum_{n=1}^{\infty} b_n z^n} \in A_1$ where b_n's are complex and

$$\sum_{n=1}^{\infty}\left[|\lambda(1-n)-1| + |\lambda(1-n)+1|\right]|b_n| < |\lambda+\mu+1| - |\lambda+\mu-1| \qquad \text{….(8)}$$

for λ,μ in ***C*** and at least one of them is nonzero, then

$$\operatorname{Re}\left[\lambda\frac{zf'(z)}{f(z)} + \mu\frac{f(z)}{z}\right] > 0, z \in U. \qquad \text{……….(9)}$$

Proof. For $f(z) = \psi(g) = z/g(z) = z/(1+\sum_{n=1}^{\infty} b_n z^n), z \in U,$ we have

$$\left|\frac{\lambda\frac{zf'(z)}{f(z)} + \mu\frac{f(z)}{z} - 1}{\lambda\frac{zf'(z)}{f(z)} + \mu\frac{f(z)}{z} + 1}\right| = \left|\frac{\lambda\frac{g(z)-zg'(z)}{g(z)} + \frac{\mu}{g(z)} - 1}{\lambda\frac{g(z)-zg'(z)}{g(z)} + \frac{\mu}{g(z)} + 1}\right|$$

$$=\left|\frac{\lambda(g(z)-zg'(z))+\mu-g(z)}{\lambda(g(z)-zg'(z))+\mu+g(z)}\right|$$

$$=\left|\frac{\lambda+\mu-1+\sum_{n=1}^{\infty}(\lambda(1-n)-1)b_n z^n}{\lambda+\mu+1+\sum_{n=1}^{\infty}(\lambda(1-n)+1)b_n z^n}\right|$$

$$\leq\frac{|\lambda+\mu-1|+\sum_{n=1}^{\infty}|\lambda(1-n)-1||b_n|}{|\lambda+\mu+1|-\sum_{n=1}^{\infty}|\lambda(1-n)+1||b_n|} \quad \ldots\ldots(10)$$

Thus,

$$\left|\frac{\lambda\frac{zf'(z)}{f(z)}+\mu\frac{f(z)}{z}-1}{\lambda\frac{zf'(z)}{f(z)}+\mu\frac{f(z)}{z}+1}\right|<1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

by the conditions (8) and (10). Now the inequality (11) gives the inequality (9).
By taking λ=1 and μ=0 in Theorem2 the next result is obtained.

Corollary1 (Reade et al[4]) If $\psi(g) = z/(1+\sum_{n=1}^{\infty} b_n z^n) \in A_1$ b_n's are complex and

$$|b_1|+\sum_{n=2}^{\infty}(n-1)|b_n|\leq 1 \quad \text{then} \quad f\in S^*.$$

By taking λ=0 and μ=1 in Theorem2 the next result is obtained, which is the same as the assertion of Part(i) of (3) for α=0.

Corollary2 If $\psi(g) = z/(1+\sum_{n=1}^{\infty}|b_n|z^n) \in A_1$, b_n's are complex and $\sum_{n=1}^{\infty}|b_n|\leq 1$ then $f\in B(0)$.

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