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PRE A*-ALGEBRAS ON TOPOLOGICAL SPACE

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Abstract:

In this paper we introduce a concept of Topological Pre A*- Algebras and prove If a Topological pre A*-Algebra A is T2 Space then it is a Hausdroff Space. A is Topological Pre A*-Algebra and I is closed ideal of A then A/I is a Topological Pre A*-Algebra and Every maximal ideal Q of a Topological Pre A*-Algebra A is closed.

Key words: Pre A*-Algebra, Compact, Pre A*-Ideal.

1 . PRELIMINARIES

1.1 Definition: An algebra $(A, \wedge, \vee, (-)^{\sim})$ satisfying

(a) $x^{\sim\sim} = x$,for all $x \in A$,

(b) $x \wedge x = x$, for all $x \in A$,

(c) $x \wedge y = y \wedge x$, for all $x, y \in A$,

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}$, for all $x, y \in A$,

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$, for all $x,y,z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$,for all $x,y,z \in A$,

(g) $x \wedge y = x \wedge (x^{\sim} \wedge y)$, for all $x,y,z \in A$ is called a Pre A*-algebra.

1.2 Example: 3 = {0, 1, 2} with operations A, □, □, (–) ˜ defined below is a Pre A*-algebra

^	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

^	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

X	x~
0	1
1	0
2	2

Note that (2, □, □, (–) ˜) is a Boolean algebra. So every Boolean algebra is a Pre A*-Algebra

1.3 Lemma: Let A be a pre A*-Algebra and a □ A be an identity for □, then a˜ is an identity for □, a unique if it exists, and is denoted by 1 and a˜ by 0.i.e

(i) a □ x = x for all x □ A (ii) a˜ □ x = x for all x □ A

1.4 Lemma: Let A be a pre A*- Algebra with 1 and 0 and let x, y □ A

(i) If x □ y = 0, implies x = 0. (ii) If x □ y =1, implies x □ x˜ = 1.

1.5 Definition: Algebra (A, □, □, (-) ˜) satisfies

(i) Pre A*- Algebra (A, □, □, (-) ˜).

(ii) A Topological space (A, τ).

(iii)The operations □, □, (-) ˜ are continuous with respect to the Topology τ, is called a Topological Pre A*-Algebra.

1.6 Example: Every Pre A*- Algebra of continuous functions with point wise convergence Topology is a Topological Pre A*-Algebra.

1.7 Note: Here after A stands for a topological pre A*-Algebra.

2. Main Results

2.1 Theorem: The following hold in A

(a)If x, y $\in$ A, then for every nbd X of x□ y, there exists W, U of x, y respectively such that W □ U $\subseteq$ X.

(b)If x, y $\in$ A, then for every nbd X of x □ y, there exists W, U of x, y respectively such that W□U $\subseteq$ X.

(c)If x $\in$ A, then for every nbd X of a˜, □ a nbd W of x , such that W˜$\subseteq$ X.

2.2 Theorem: Suppose L, M are subsets of A, then

(1) LM, L+M, L□M, L□M, L˜ are compact sets, whenever L, M are compact sets.

(2) LM, L+M, L□M, L□M, L˜ are connected sets, whenever K, S are connected sets.

Proof: **(**1) Since continuous image of a compact set is a compact.

•: A × A⟶A is continuous, L,M are compact sets

i.e., L× M is a compact in A×A, so • (L×M) =LM is a compact in A.

‖y L + M,L□M,L□M,L˜ are compact sets.

(2)Since continuous image of connected sets is connected

•: A×A⟶ A is continuous, L× M is connected in A×A.

So • (L×M) =LM is connected set

‖y L+M,L□M,L□M,L˜ are connected sets.

2.3 Theorem : The union of all connected subsets contain 0 is a Topological Sub pre A*-Algebra.

Proof: Suppose {P*i* /*i* ∈ I} is a class of all connected sets containing 0.

Let P = U Pi contain 0.
i∈ I

∵ 0∈ P ⇒0 ∈ P*i* for some *i* ∈ I ⇒1 – 0 ∈ P*i* □⇒1 ∈P*i*□
,

∴ P□*i* is also connected. (∵ P*i* is connected)

∴1 ∈P. Let a ∈P ⇒a ∈K *i* for some *i* ⇒– a ∈ K □*i* ⇒ – a ∈ P.

a ∈ P ⇒– a∈ P

Suppose a, b ∈P⇒ a ∈ P*i*, b ∈ P*j* ⇒ a + b ∈ P*i* + P*j*

∴ a + b ∈ P (∵P*i* + P*j* is connected)

∴ P is a topological sub Pre A*-Algebra of A.

2.5 Theorem: Every Topological Pre A*-Algebra is a homogeneous algebra.

Proof: Let A be a pre A*-Algebra. Let c=q-p Define a function f:A⟶A by f(x) = (q-p)+x. Then f is continuous and f(p) = c + p = q- p + p = q.

⇒ f (p) = q.

2.6 Theorem: If a Topological pre A*-Algebra A is T2 Space, then it is a Hausdroff Space.

Proof: Let A be a pre A*-Algebra. Let x, y ∈ A and x≠y. Since A is T2- Space

⟹□ Nx, Ny nbds of x, y respectively ∋ x∉ Ny, y∉Nx

Suppose Nx∩Ny ≠ ∅

Let V= Nx ∩ Ny

Let z∈ V and z≠0 Then U = v-z is a nbd of 0

Let Ux = U+ x, Uy=U+ y, then Ux, Uy are nbds of x, y respectively and Ux ∩ Uy are nbds of x, y respectively and Ux ∩ Uy =∅ ∴A isHausdroff space.

2.7 Definition: Let (A, □, □, (−) ˜, 0, 1) be a pre A*-Algebra. A non empty set I of A is said to be a Pre A*-Ideal of A if

(i) a, b∈ I⟹ a□b∈ I

(ii) a∈I, b∈A⟹ a□b∈ I.

2.8 Theorem: Suppose A is Topological Pre A*-Algebra and I is closed ideal of A. Then A/I= {Ia /a∈A} is a Topological Pre A*-Algebra.

Proof: Define □, □, (−) ˜, 0, 1 in A/I as follows

(i)Ia □ Ib = Ia □ b (ii) Ia □ Ib = Ia □ b.(iii)(Ia)˜ = Ia˜

(iv)0=Io, 1=I1 Then (A/I,□,□,(−)˜,Io,I1) is a Pre A*-Algebra Define f: A→A/I by f (a) =Ia , where a∈ I.

Suppose a=b ⟹ Ia =Ib. ∴ f is well defined and clearly f is surjective.

Clearly f: A→A/I is a Pre A*-Homomorphism. ∴f: A→A/I is an Pre A*-epimorphism. Suppose τ1 = {f (u) /u∈ τ}.Since I is closed, τ1 is a Topology on A/I for which □, □, (−) ˜ in A/I are continuous. ∴ A/I is a Topological Pre A*-Algebra.

2.9 Theorem: If I is an Ideal in the Topological Pre A*-Algebra A, then □ is also an ideal in A

Proof: Suppose I is an ideal in the Topological Pre A*-Algebra A.

□= {a∈A/ Every nbd of a intersects I}.

Claim: □ is an ideal.

Let a, b∈ □⟹ every nbd of a and every nbd of b interest I.

Let W be a nbd of a□ b. Then □ nbds U, V of a, b ∋ U□V⊆ W.

∵ U,V intersect I, So U ∩ V intersect I, So W intersect I.So a □ b∈□

Suppose a∈ □, b∈ A. ∵ a ∈ □⟹ Every nbd of a intersect I.

Let W be the nbd of a□ b ⟹□ nbd U of ab ∋ U⊆ W.

⟹□ nbd V, G of a,b ∋ V□G⊆ U ⟹V□ G ⊆ U.

i.e V□G ⊆ U ⊆ W.Since V intersect I so V∩ G

∴ W intersect I, ∵ V∩ G⊆ G ∴ a □ b ∈ □ ∴ □ is an ideal.

2.10 Theorem: Every maximal ideal Q of a Topological Pre A*-Algebra A is closed.

Proof: Clearly Q$\subseteq \bar{Q}$ But $\bar{Q}$ is an ideal of A.$\therefore$ Q=$\bar{Q}$,$\because$ Q is maximal. $\therefore$Q is closed.

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