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EFFECT OF HEAT SOURCE ON STEADY MHD FLOW AND HEAT TRANSFER OVER A HEATED STRETCHING SURFACE

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Abstract

The paper deals with the effect of heat source on steady flow of a viscous incompressible fluid over a heated stretching sheet with temperature dependent viscosity. A magnetic field of uniform strength is applied normal to the flow. In order to establish a finite condition $z \to 1$ instead of an infinite boundary condition, the governing equations in non-dimensional form are transformed to new system of co-ordinates, in which the equation of motion is a non-linear ordinary differential equation that is linearised by Quassi-linearisation method. Then they are transformed to system of linear equations using finite difference formulae for which a numerical code is executed using C-program.

The results for the velocity, temperature, Skin friction and Heat transfer are discussed and analysed.From this study, it is found that the temperature and velocity of the fluid increase in the presence heat source.

Keywords: Steady flow, heated stretching sheet, heat source, Quassi-linearization and finite difference methods.

1 Introduction

The transfer of heat, mass and momentum in the laminar boundary layer flow on a heated stretching sheet are considerable important from both theoretical practical point of view. Such situations may arise often in polymer processing industry, liquid thin-film development and other related surface flows. Sakiadis [1] studied first the boundary layer flow over a continuous solid surface moving in its own plane with constant speed. He showed that the characteristics of the boundary layer in this case is quite different from that of the blassius flow due to entrainment of the ambient fluid. Erickson et al [2] investigated a similar problem in which the transverse velocity at the moving surface is non-zero, taking account of the heat and mass transfer in the boundary layer. Investigations of this type are important due their relevance to the problem of a polymer sheet extruded continuously from a dye. A tacit assumption is being made that the sheet is inextensible. In polymer industry it is necessary to tackle the boundary layer flow over a stretching sheet, McCormack and Crane [8]. Gupta and Gupta [5] carried out the analysis of momentum, heat and mass transfer in the boundary layer over a stretching sheet, subjected to suction or blowing. Radwan *et al* [9] examined the mass transfer over a stretching surface with variable concentration in a transverse magnetic field. In all the cases mentioned above the viscosity of the fluid was assumed uniform in the flow region. Jang *et al* [7] studied the rate of temperature dependent viscosity in the flow and vortex instability of a heated horizontal free convection boundary layer flow. Ioan Pop *et al [6*] analyzed the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate. Lai *et al* [3] studied the effect of variable viscosity on convective heat transfer along a vertical surface in saturated porous medium.

Several investigators have studied different dimensions of the boundary-layer flow of electrically conducting fluid and heat transfer due to stretching sheet in the presence of a transverse magnetic field. The flow of an electrically conducting fluid past stretching sheet under the effect of a magnetic field has attracted the attention of many researchers in view of its wide applications in many engineering problems such as magneto hydrodynamic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extraction, and the boundary layer control in the field of aerodynamics

Samad and Mohebujjaman [11] studied a steady-state two dimensional magneto hydrodynamic heat and mass transfer free convection flow along a vertical stretching sheet in the presence of a magnetic field with heat generation. Fadzilah et al.[12] discussed the steady magneto-hydrodynamic boundary-layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet with an induced magnetic field. Ishak [13,14] studied the steady MHD boundary-layer flow and heat transfer due to a stretching sheet. Mixed convection boundary layer in the stagnation point flow to-wards stretching sheet was studied by Ishak et al. [15].Recently Unsteady MHD boundary-layer flow and heat transfer over a stretching sheet in the presence of heat source or sink is studied by Ibrahim et al. [16]. Lahiri *et al* [10] analyzed the effects of transverse magnetic field on the momentum and heat transfer characteristics in the boundary layer of an incompressible fluid flow over a stretching sheet when viscosity of the fluid depends on temperature. Lahiri *et al* in their research paper, applied shooting method to solve the governing flow of a fluid.

In most of the earlier steady flow investigations there seems to be no significant consideration of the effect of heat source, which plays a vital role in maintaining heat transfer at desired level in the applications of Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles. So in the present paper a numerical attempt is made to analyze effect of heat source on steady flow of a viscous incompressible fluid over a heated stretching sheet with temperature dependent viscosity. Using appropriate similarity transformations, the momentum and energy equations are reduced to ordinary differential equations, in which the equation of motion is a non-linear ordinary differential equation, which is linearized by quassi-linearization method. In order to obtain the approximate solution and to describe the physics of the problem, finite difference method is employed to solve the transformed equations.

2 Mathematical Formulation:

Steady two-dimensional flow of a viscous incompressible, electrically conducting fluid over a heated stretching sheet is considered. The motion of the fluid is being caused solely by the surface` which is moving horizontally with a speed proportional to the distance from the origin (x=0). Additionally, the viscosity of the fluid is assumed to be dependent on the temperature. A magnetic field of uniform strength is applied normal to the flow. The continuity, momentum and energy equations governing such a flow in the boundary layer when subjected to an magnetic field of strength β_0 (Ferraro *et al* [4]) are written, as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q(T-T_\infty)}{\rho C_p} \quad (3)$$

Where u and v are the components of velocity, respectively in the x and y directions, T is the temperature, α is the coefficient of thermal diffusivity, ρ is the fluid density, σ is the conductivity of the fluid and μ is the coefficient of fluid viscosity. The boundary conditions are given by

$$u=cx,\quad v=0,\quad T=T_{wt}\quad at\quad y=0 \quad (4)$$

$$u\to 0,\quad T\to T_\infty \quad as\ y\to\infty \quad (5)$$

Hence $c(>0)$ is a constant, T_w is the uniform wall temperature and T_∞ is the free-stream Temperature. We now introduce the following relations for u, v and θ as

$$u=\frac{\partial \psi}{\partial y},\qquad v=-\frac{\partial \psi}{\partial x}, \quad (6)$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \quad (7)$$

Where ψ is the stream function. The temperature dependent –viscosity is given by (Bird et al 1960)

$$\mu=\mu^* e^{a(T_w-T)}, \quad (8)$$

Where μ^* is the reference viscosity and a is a constant.

Using the relations (6), (7) and (8) in the equations (2.2) to (2.3) we obtain the following

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=-Av^* e^{A(T_w-T)}\frac{\partial \theta}{\partial y}\frac{\partial^2 \psi}{\partial y^2}+v^* e^{A(1-\theta)}\frac{\partial^3 \psi}{\partial y^3}-M\frac{\partial \psi}{\partial y} \quad (9)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2\theta}{\partial y^2}+\frac{Q\theta}{\rho C_p} \qquad (10)$$

Where $\alpha=\frac{k}{\rho C_p}, v^*=\frac{\mu^*}{\rho}, M=\frac{\sigma B_0^2}{\rho c}$ and c has a dimension (1/ time).

Using the following similarity transformations

$$\psi=(c\,v^*)^{\frac{1}{2}}x\,f(\eta),\ \eta=\left(\frac{c}{v^*}\right)^{\frac{1}{2}}y\ , \qquad (11)$$

in the equations (2.9) and (2.10), we get

$$\left(\frac{df}{d\eta}\right)^2-f\frac{d^2f}{d\eta^2}+Ae^{A(1-\theta)}\frac{d\theta}{d\eta}\frac{d^2f}{d\eta^2}=e^{A(1-\theta)}\frac{d^3f}{d\eta^3}-M\frac{df}{d\eta} \qquad (12)$$

$$\frac{d^2\theta}{d\eta^2}+\Pr f\frac{d\theta}{d\eta}+S\theta=0 \qquad (13)$$

where $\Pr=\frac{\mu\,c_p}{k}, S=\frac{Q\upsilon}{k}$

The corresponding boundary conditions reduced to

$$f(0)=0,\qquad f'(0)=1,\qquad \theta(0)=1 \qquad (14)$$

$$f'(\infty)=0,\qquad \theta(\infty)=0 \qquad (15)$$

Here, M and $\Pr$ are called the non-dimensional Magnetic parameter and Prandtl numbers, respectively.

3 Method of solution

Quasi- linearization of all non-linear terms in equation (12) gives the following

$$B_2\frac{d^3f}{d\eta^3}+\left(F-B_1\right)\frac{d^2f}{d\eta^2}-\left(M+2F'\right)\frac{df}{d\eta}+F''f=F\,F''-\left(F'\right)^2 \qquad (16)$$

Where $F'=\frac{F(i+1)-F(i)}{h}$, $F''=\frac{F(i+1)-2F(i)+F(i-1)}{h^2}$, they are called finite difference approximations of F' and F''. Here F is assumed to be a known function.

In order to establish a finite condition $z \to 1$ instead of an infinite boundary condition $\eta \to \infty$, in (18), we transform the equations (13) and (16) to a new system of co-ordinates. So employing the transformation $z = 1 - e^{-b\eta}$ on the equations (13) and (16), we get

$$B_2 Q^2 \frac{d^3 f}{dz^3} + F_1 \frac{d^2 f}{dz^2} + F_2 \frac{df}{dz} + F_3 f = F_4 \tag{17}$$

$$Q \frac{d^2\theta}{dz^2} + (P_r f - b)\frac{d\theta}{dz} + \frac{S}{Q}\theta = 0 \tag{18}$$

Where

b = Constant, $B_1 = A e^{A(1-\theta)} \theta'$

$B_2 = e^{A(1-\theta)}$, $Q = b(1-z)$

$F_1 = (F - B_1) - 3b B_2 Q$, $F_2 = b^2 B_2 - b(F - B_1) - 2QF' - M$

$F_3 = QF'' - bF'$, $F_4 = F(QF'' - bF') - Q(F')^2$

In $z = 1 - e^{-b\eta}$, constant b is used as a scaling factor to provide an optimum distribution at the nodal points across the boundary layer.

The corresponding boundary conditions are reduced to:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \tag{19}$$

$$f'(1) = 0 \quad \theta(1) = 0$$

Substitution of the following finite difference formulae

$$\frac{d^2 g}{dz^2} = \frac{g(i+1) - g(i)}{h}$$

$$\frac{d^2 g}{dz^2} = \frac{g(i+1) - 2g(i) + g(i-1)}{h^2} \tag{20}$$

$$\frac{d^3 g}{dz^3} = \frac{g(i+2) - 3g(i+1) + 3g(i) - g(i-1)}{h^3}$$

where g stands for f and $\boldsymbol{\theta}$

Applying the finite difference formulae (20) on the equations (17) and (18), we obtain the following system of equations

$$A_1[i]\ f[i+2] + A_2[i]\ f[i+1] + A_3[i]\ f[i] + A_4[i]\ f[i-1] = A_5[i] \tag{21}$$

$$H_1[i]\theta[i+1] - H_2[i]\,\theta[i] + Q\theta[i-1] = 0 \tag{22}$$

Where

$A_1[i] = B_2[i]Q^2[i]$,

$A_2[i] = F_1[i]h + F_2[i]h^2 - 3A_1[i]$

$A_3[i] = 3A_1[i] - 2F_1[i]h - F_2[i]h^2 + F_3[i]\,h^3$

$A_4[i] = F_1[i]h - A_1[i]$, $\quad A_5[i] = F_4[i]h^3$

$H_1[i] = Q + (P_r f[i] - b)h$, $\quad H_2[i] = 2Q[i] + (P_r f[i] - b)h + \dfrac{S}{Q}$.

In the above and h represents the mesh size in z-direction.

Equations **(21)** and (**22**) with corresponding boundary conditions have been solved by using Gauss-seidel iteration method**,** for which simulation is carried out by coding in C-Program. In the above system f is considered as the $(n)^{th}$ order iterative solutions and F is the $(n-1)^{th}$ order solutions. After each cycle of iteration the convergence check is performed, i.e the tolerance set at 10^{-6}, i.e., $|F - f| < 10^{-6}$ is satisfied at all points, then f is considered as convergent solution. Otherwise f becomes the new F and another cycle of iteration is carried out.

Skin friction coefficient and Nusselt number

As the local skin-friction coefficient is an important characteristic of the boundary layer flow, as defined by

$$C_f = \frac{-2f''(0)}{(cx^2 v^*)^{\frac{1}{2}}}$$

We calculate $-f''(0)$ and $-\theta'(0)$ taking various values of heat source parameter S

RESULTS AND DISCUSSION:

In order to obtain the approximate solution and to describe the physics of the problem, the present non-linear boundary value problem is solved numerically using finite difference method.. In the current section, the effects of different flow parameters such as heat source parameter (S), magnetic parameter (M) and temperature dependent viscosity (A) on velocity, temperature are discussed and analyzed.

Figure (1) and (3) display the effect of heat source on temperature and velocity profiles respectively. It is observed that an increase in z leads to decrease in the velocity and temperature and also it is observed that the velocity vanishes at $Z = 1.0$. Further it is interesting to note that the velocity and temperature increase as the value of heat source parameter S increases. This result qualitatively agrees with expectation since the effect of heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid and also increasing its velocity.

Figs (2) and (4) show s the effect of magnetic parameter **M** on temperature and velocity field u respectively. It is noted from figure that an increase in M leads to decrease in the velocity. The presence of magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow when the magnetic field is applied normal to the fluid flow. This type of resistive force tends to slow down the flow field. Further it is also observed that temperature increases in the presence of magnetic field. Further from figure (4), it is noted that temperature of the fluid increases as the value of M increases. From figure (5), It is observed that the velocity of the fluid decreses with the increasing values of temperature dependent viscosity parameter A.

The impact of the physical parameter i.e heat source parameter S on local skin friction and local rate of heat transfer is shown in table 1. For the case of a fixed Prandtl number Pr=0.71 and temperature dependent viscosity $A = 1$, the local rate of heat transfer and the local skin friction coefficient and i.e $-\theta'(0)$ **and** $-f''(0)$ increase as the value of S increases.

CONCLUSIONS:

- The temperature and velocity of the fluid increase in the presence heat source. This due to the fact that effect of heat generation is to increase the rate of heat transport to the fluid thereby increasing the temperature and its velocity of the fluid

Nomenclature:

u, v	Components of velocity in the x and y directions
μ	Dynamic viscosity.
ρ	Density of the fluid
α	The coefficient of thermal diffusivity
ψ	Stream function
θ	Non-dimensional temperature
T	Temperature of the fluid.
T_w	Temperature of the wall of the stream
T_∞	Free stream temperature.
μ^*	Reference viscosity.
v^*	Reference kinematics viscosity.
σ	Conductivity of the fluid.
a	Constant in variable
A	Temperature dependent viscosity
η	Similarity variable
P_r	Prandtl number.
f	Non-dimensional stream function defined in (2.11)
M	Non-dimensional magnetic parameter.
C_f	Skin –friction coefficient.

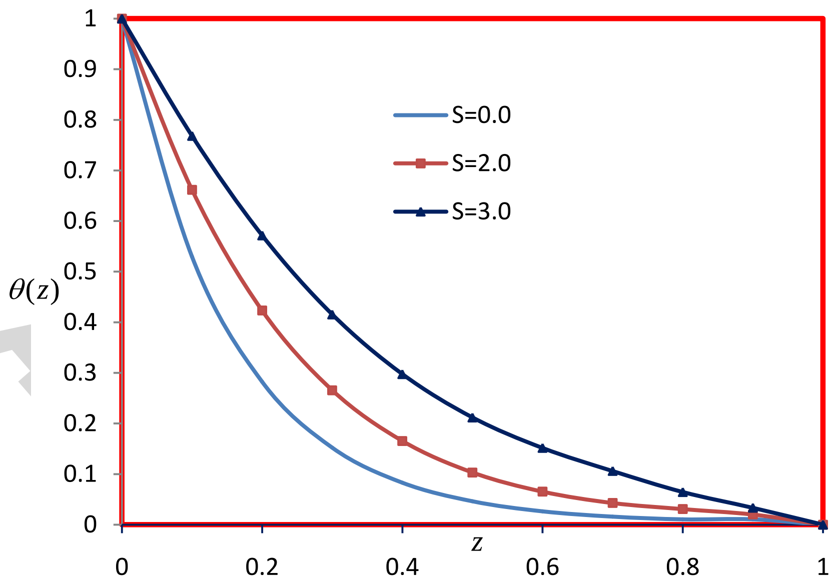

Fig1: Effect of heat source on temperature field θ
(A=1.0, M=1.0 and P

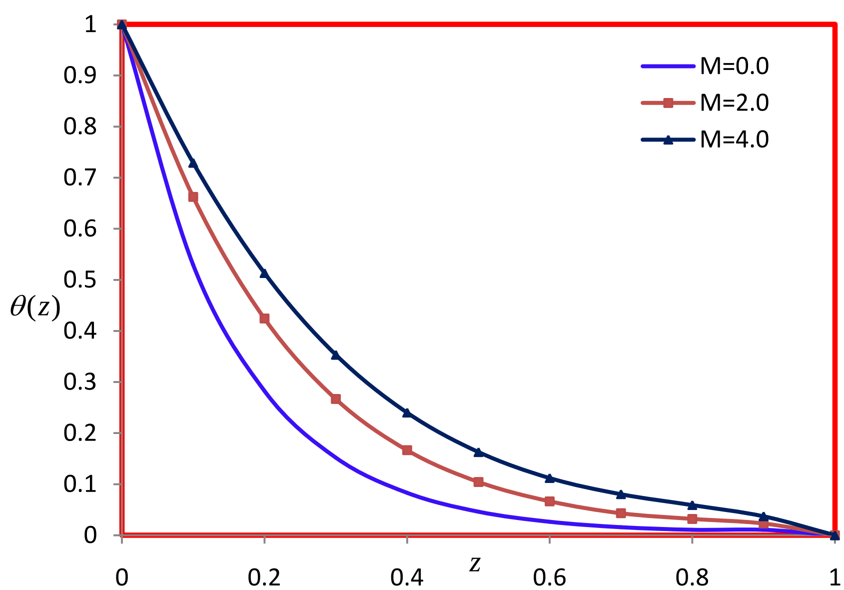

Fig 2: Effect of Magnetic parameter M on temperature field θ
(A=1.0, S=1.0 and Pr =0.71)

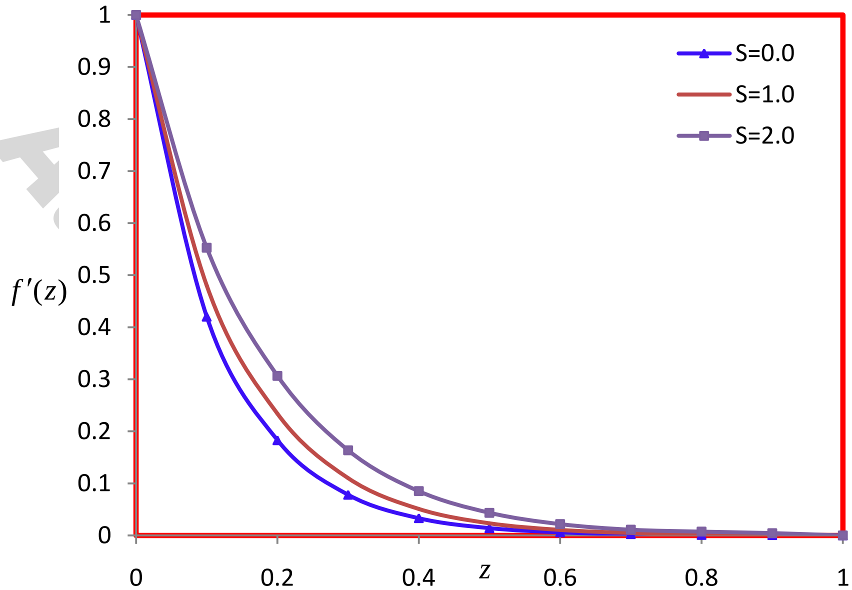

Fig 3: Effect of heat source S on velocity $f'(z)$
(**A**=1.0, M=1.0 and Pr =0.71)

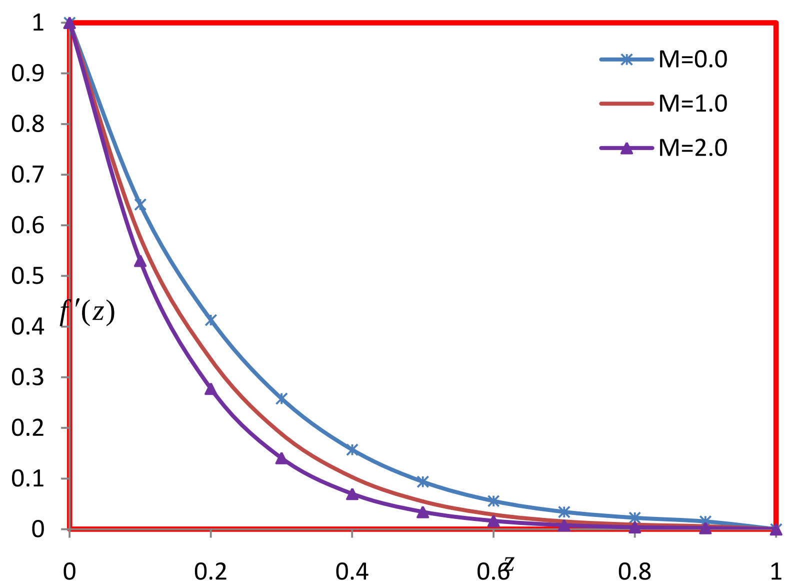

Fig 4: Effect of Magnetic parameter M on velocity $f'(z)$
(**A**=1.0, S=1.0 and Pr =0.71)

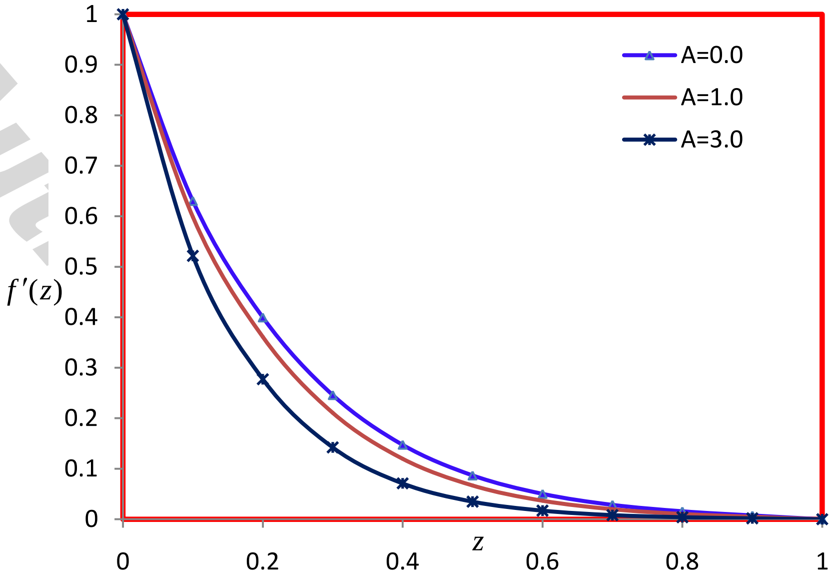

Fig 5: Effect of temperature dependent viscosity A on velocity $f'(z)$
(M=1.0, S=1.0 and Pr =0.71)

Table1: Skin-friction and Nusselt umber
(M=1.0, A=1.0 and Pr=0.71)

S	$-f''(0)$	$-\theta'(0)$
1.0	11.5974	9.4080
2.0	10.3639	6.7665
3.0	8.9460	4.7636

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