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SECOND ORDER VISCO-ELASTIC FLUID FLOW THROUGH A POROUS MEDIUM WITH SMALL SUCTION REYNOLD'S NUMBER BETWEEN TWO HORIZONTAL PERMEABLE PLATES IN RELATIVE MOTION

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Abstract

The present paper deals with the "Steady flow of a Noll's second order fluid through a porous medium between two horizontal permeable plates in relative motion with small suction Reynold's number". The flow through the porous medium satisfies the general momentum equation proposed by Yama Moto and Yoshida and the bounding plates are permeable allowing equal suction and injection through them. A perturbation technique has been adopted with the suction Reynold's number (R) as perturbation parameter to obtain an approximate solution of an ill-posed problem. The velocity field is obtained analytically from which the flow rate, mean velocity in the channel and the drag forces on two bounding plates are obtained.

The special cases of flow in (i) Wide channel (ii) Narrow channel (iii) Large porosity parameter and (iv) Small porosity parameter have been discussed and the results have been drawn from graphical illustrations.

Key words: Steady flow, Visco-elastic fluid, porous medium, perturbation technique

Introduction:

The study of flows through porous medium have been a topic of considerable research activity by both theoretical and experimental investigators for the last one and half century because of its wide applications in diverse field of science, engineering and technology, geophysics, soil sciences, agriculture engineering, biotechnology and so on.

The momentum equation for the flows through a channelised porous medium is proposed by Yama Moto and Yoshida [13] which takes into account the inertia effects and the internal stress effects as well. The fluid under the consideration is a second order visco–elastic fluid of the Coleman and Noll[3] type constitutive equation, which is expressed as a polynomial in the rate of deformation tensor and the acceleration gradient tensor.

The laminar flow between the porous channel has been initiated as early as in 1953 by Berman [2] followed by Kapoor et. al. [4]. Problems concerning second order visco-elastic fluids were extensively studied earlier by several investigators that would include Pattabhi Ramacharyulu[7], Ramana Murthy [11],Raju and Devanathan[10].

Flows through porous medium adopting the general momentum equation of Yama Moto and Yoshida [13] was also studied earlier by several other investigators that will include Pattabhiramacharyulu[8] ,Narasimhacharyulu[5], Raghavacharyulu[9], and Appala Raju[1]. A variety of flows through porous media in different geometries have been discussed in the references cited.

The present paper deals with the second order visco-elastic fluid through a generalized porous medium between two permeable horizontal boundaries in relative motion. A constant suction and an equal injection of the fluid is assumed. An approximate solution for small suction Reynold's number for an ill-posed problem (a third order differential equation with only two boundary conditions) is obtained by employing perturbation technique with the suction Reynold's number(R) as the perturbation parameter.

Velocity distribution is obtained analytically upto the first order R. Further the flow rate, mean velocity in the channel and drag forces on the two plates have been computed and their variations are illustrated graphically. Flows for A:(i) Wide channel (ii) Narrow channel and B:(i) Large porosity (ii) Small porosity are also examined . A boundary layer effect, the thickness of the boundary layer is $O(1/\alpha)$ is noticed near the plates and the core region is almost plug type as one goes deep into the channel.

Mathematical formulation:

The momentum equation for the fluid flow through a generalized porous medium as suggested by Yama Moto , and Iwamura[12] is given by

$$\rho \frac{d\bar{q}}{dt} = divS - \frac{\mu}{k}\bar{q}$$

together with the continuity equation for incompressible flow

div $\bar{q} = 0$

In this

ρ =fluid density (constant)

$\bar{q}$ =fluid velocity

s= stress tensor

μ = the coefficient of Newtonian viscosity

k= coefficient of permeability (porosity)

The stress strain rate relations of a second order visco-elastic fluid Noll and Coleman[3] type are

$$S = -PI + \phi_1 A^{(1)} + \phi_2 A^{(1)^2} + \phi_3 A^{(2)} \qquad \text{-------------------} \qquad (1)$$

where $A_{ij}^{(1)} = 2E_{ij} = V_{i,j} + V_{j,i}$

and

$$A_{ij}^{(2)} = \frac{\partial}{\partial t} A^{(1)}{}_{ij} + V_m{}^{(1)} A^{(1)}{}_{1j,m} + A_{m_1}{}^{(1)} V_{i,j}{}^{(1)m} + A_{mj}{}^{(1)} V_{j,i}{}^{(1)m} \qquad \text{---------- (2)}$$

where S= the stress tensor, P= Isotropic mean pressure

V_i= the velocity in the i^{th} direction

$(\phi_1, \phi_2, \phi_3) = \rho(\upsilon, \upsilon_c, \beta)$ are the material constants and ρ is the fluid density.It can be noted that υ the classical viscosity, υ_c the cross viscosity and β are visco-elasticity coefficients respectively.

In this paper, a steady flow of a second order visco elastic fluid flowing through a porous medium bounded by two horizontal parallel permeable plates in relative motion is presented.

Consider a system of rectangular Cartesian coordinates $O(X,Y,Z)$ with the origin fixed on the bottom plate, X-axis parallel to the flow direction (the direction of the applied pressure gradient parallel to the plates) and Y-axis perpendicular to the plates.

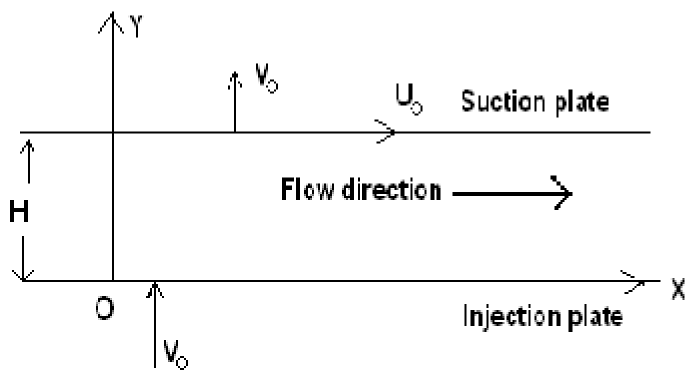

Fig.1 Flow configuration in parallel plate geometry with constant injection/suction With Suction plate in motion

With reference to this frame of reference the plates can be represented by Y=0 and Y=H. The flow is generated by a constant pressure gradient along the X-axis and the upper plate Y=H is kept moving with a velocity U_0. The two plates are permeable allowing a cross flow with a constant injection velocity v_0 at the bottom plate and equal suction at the upper plate.

The fluid velocity may be taken as

$$\bar{q} = (U(Y), V_0, 0) \qquad \text{------- (3)}$$

This choice of velocity evidently satisfies the continuity equation

$$div\bar{q} = 0 \qquad \text{------- (4)}$$

The stress tensor is given by

$$S_{XX} = S_{YY} = -P + \mu_c U^{1^2} \qquad \text{---------- (5)}$$

$$S_{XY} = S_{YX} = \mu U^{1}(Y) + \mu_2 v_0 U^{11}(Y) \qquad \text{--------------- (6)}$$

$$P_{\text{mod}} = P - \mu_c U^{'2}(Y) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

the momentum equation in the X-direction is

$$\rho v_0 \frac{dU}{dY} = -\frac{\partial}{\partial X} P_{\text{mod}} + \mu \frac{d^2U}{dY^2} + \mu_2 v_0 \frac{d^3U}{dY^3} - \frac{\mu}{k} U \qquad \text{--------- (8)}$$

with the boundary conditions (conditions of no slip) are

$$U(0) = 0 \quad \text{and} \quad U(H) = U_0 \qquad \text{---------- (9)}$$

and that in the y-direction

$$0 = 0 + \frac{\partial}{\partial Y}(-P_{\text{mod}}) - \frac{\mu}{k} v_0 \qquad \text{--------- (10)}$$

The flow is assumed to be under the constant pressure gradient $-\frac{\partial P}{\partial X} = -C_1$ i.e.

$$-\frac{\partial}{\partial X} P_{\text{mod}} = -C_1 \qquad \text{--------- (11)}$$

From the equations (7),(10) and (11) we obtain

$$P_{\text{mod}} = C_1 X - \frac{\mu}{k} v_0 Y + \mu_c U'^2(Y) + p^* \qquad \text{----------------------------------- (12)}$$

Where p* is an arbitrary constant of integration which can be interpreted as the indeterminate pressure in the fluid. Thus U(y) obtained from equations (8) and (9) would decide the pressure p but for an arbitrary constant p*.

The following non-dimensional scheme is introduced

$$\left.\begin{array}{llll} X = ax & P_{\text{mod}} = \dfrac{\mu^2 p_m}{\rho a^2} & v_0 = \dfrac{R\mu}{\rho a} & -\dfrac{\partial p_m}{\partial x} = c_1 \\ Y = ay & & \mu_2 = \beta \rho a^2 & U_0 = \dfrac{\mu u_0}{\rho a} \\ H = ah & U = \dfrac{\mu u}{\rho a} & k = \dfrac{a^2}{\alpha^2} & \end{array}\right\} \qquad \ldots\ldots\ldots(13)$$

In the above scheme R= Suction Reynold's number and β = Visco-elastic parameter.

In terms of the above non-dimensional quantities, the momentum equation in the X-direction reduces to

$$\beta R \frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - \alpha^2 u = -c_1 \qquad \text{----------- (14)}$$

With boundary conditions $u(0) = 0$ and $u(h) = u_0$ ----------------- (15)

It is noticed that the linear equation (14) is of third order with only two end conditions (15). This makes the problem ill-posed encountering the problem of a third order differential equation with two boundary conditions, while it is well-posed when there is no cross flow (no suction/injection i.e. when R=0) and/or when the fluid is not visco-elastic (i.e.β=0).

However we attempt to present an approximate solution of this equation for small values of the suction/injection Reynold's number R.

Let us assume that the flow velocity for small R in the form

$$u = u^{(0)} + Ru^{(1)} + R^2u^{(2)} + ---- \qquad \text{--------- (16)}$$

Substituting this in the equation (14) and the boundary conditions (15) and collecting like powers of R, we get the equations for the different orders of approximation for 'u', which can be solved.

Basic/zeroth order approximation

In this the terms contained in R^0 are collected and obtain

$$u^{(0)^{11}} - \alpha^2 u^{(0)} = -c_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(17)}$$

With boundary conditions $u^{(0)}(0)$ and $u^{(0)}(h) = u_0$ ……………..(18)

These equations yield the velocity

$$u^{(0)}(y) = \frac{c_1}{\alpha^2}\left[1 - \frac{\sinh\alpha(h-y)}{\sinh\alpha h}\right] + \left(u_0 - \frac{c_1}{\alpha^2}\right)\frac{\sinh\alpha y}{\sinh\alpha h} \qquad \ldots\ldots\ldots \text{(19)}$$

This velocity is the same as that of the flow in the case of impermeable walls.

At this state, we get the **flow-rate;**

$$q^{(0)} = \int_0^h u^{(0)} dy = \frac{c_1}{\alpha^2}\left[h - \frac{2\tanh\frac{\alpha h}{2}}{\alpha}\right] + u_0 \frac{\tanh\frac{\alpha h}{2}}{\alpha} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(20)}$$

and the mean velocity;

$$\bar{u}^{(0)} = \frac{q^{(0)}}{h} = \frac{c_1}{h\alpha^2}\left[h - \frac{2\tanh\frac{\alpha h}{2}}{\alpha}\right] + u_0 \frac{\tanh\frac{\alpha h}{2}}{\alpha} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(21)}$$

First approximation

Collecting the terms with R we get the equation for $u^{(1)}$

$$u^{(1)''} - \alpha^2 u^{(1)} = Ru^{(0)'} - u^{(0)'''} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(22)}$$

with the boundary conditions $u^{(1)}(0)=0$ and $u^{(1)}(h)=0$ ……………(23)

solving the equation (22), we get

$$u^{(1)}(y) = \frac{(R - \alpha^2\beta)}{2\sinh\alpha h}\left[\left(u_0 - \frac{c_1}{\alpha^2}\right)(y-h)\sinh\alpha y - \frac{c_1}{\alpha^2} y \sinh\alpha(h-y)\right] \qquad \ldots\ldots\ldots\text{(24)}$$

the flow rate :

$$q^{(1)} = \int_0^h u^{(1)} dy = \frac{u_0 R\left(R-\alpha^2\beta\right)}{2\alpha \sinh \alpha h}\left\{h - \frac{\sinh \alpha h}{\alpha}\right\} \quad \text{.................(25)}$$

and mean velocity:

$$\bar{u}^{(1)} = \frac{q^{(1)}}{h} = \frac{u_0 R\left(R-\alpha^2\beta\right)}{2h\alpha \sinh \alpha h}\left\{h - \frac{\sinh \alpha h}{\alpha}\right\} \quad \text{....................(26)}$$

The net velocity up to the first order of R is

$$u = u^{(0)} + Ru^{(1)} \quad \text{..................................(27)}$$

$$= \frac{c_1}{\alpha^2}\left[1 - \frac{\sinh \alpha (h-y)}{\sinh \alpha h}\right] + \left(u_0 - \frac{c_1}{\alpha^2}\right)\frac{\sinh \alpha y}{\sinh \alpha h} + \frac{R\left(R-\alpha^2\beta\right)}{2\sinh \alpha h}\left[\left(u_0 - \frac{c_1}{\alpha^2}\right)(y-h)\sinh \alpha y - \frac{c_1}{\alpha^2} y \sinh \alpha (h-y)\right] \quad \text{........ (28)}$$

Then the flow rate :

$$q = \int_0^h u dy = \frac{c_1}{\alpha^2}\left[h - \frac{2\tanh\frac{\alpha h}{2}}{\alpha}\right] + u_0 \frac{\tanh\frac{\alpha h}{2}}{\alpha} + \frac{u_0 R^2\left(R-\alpha^2\beta\right)}{2\alpha \sinh \alpha h}\left\{h - \frac{\sinh \alpha h}{\alpha}\right\} \quad \text{... (29)}$$

and the mean velocity:

$$= \frac{q}{h} = \frac{c_1}{h\alpha^2}\left[h - \frac{2\tanh\frac{\alpha h}{2}}{\alpha}\right] + u_0 \frac{\tanh\frac{\alpha h}{2}}{\alpha h} + \frac{u_0 R^2\left(R-\alpha^2\beta\right)}{2\alpha h \sinh \alpha h}\left\{h - \frac{\sinh \alpha h}{\alpha}\right\} \quad \text{.... (30)}$$

The Drag force/Shear Stress on the bottom plate is

$$\left[\frac{du}{dy} + \beta R \frac{d^2 w}{dy^2}\right]_{y=0} = \frac{c_1}{\alpha^2}\left[\coth \alpha h - \frac{1}{\sinh \alpha h} - \beta R \alpha\right] + \frac{u_0 \alpha}{\sinh \alpha h} + \frac{R\left(R-\alpha^2\beta\right)}{2\sinh \alpha h}\left[\frac{-c_1}{\alpha^2}\left\{(-\alpha h + \sinh \alpha h) + 2\alpha\beta(1-\cosh \alpha h)\right\} + u_0 \alpha(-h + 2\beta R)\right] \quad \text{.. (31)}$$

The Drag force i.e. Shear Stress on the top Plate is

$$\left[\frac{du}{dy} + \beta R \frac{dw}{dy^2}\right]_{y=h} = \frac{c_1}{\alpha^2}\left[\frac{\alpha}{\sinh \alpha h} - \alpha \coth \alpha h - \beta R \alpha^2\right] + u_0\left[\alpha \coth \alpha h + \beta R \alpha^2\right] + \frac{R\left(R-\alpha^2\beta\right)}{2\sinh \alpha h}\left[-\frac{c_1}{\alpha^2}\left\{(\sinh \alpha h - \alpha h) + 2\alpha\beta(\cosh \alpha h - 1)\right\} + u_0\left(\sinh \alpha h + 2\alpha\beta R \cosh \alpha h\right)\right] \quad \text{. (32)}$$

The flow for wide channel i.e. flow for large ‘h’:

For large h $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$ and & $\tanh \alpha h \approx 1$

The net velocity up to the first order of R for wide channel is

$$u = u^{(0)} + Ru^{(1)}$$

$$= \frac{c_1}{\alpha^2}\left(1-e^{-\alpha y}\right) + \left(u_0 - \frac{c_1}{\alpha^2}\right)e^{-\alpha(h-y)} + \frac{R^2\left(R-\alpha^2\beta\right)}{2}\left[\left(u_0 - \frac{c_1}{\alpha^2}\right)(y-h)e^{-\alpha(h-y)} - \frac{c_1 y}{\alpha^2}e^{-\alpha y}\right] \quad \text{........ (33)}$$

Hence the velocity near the bottom is

$$u_b = \frac{c_1}{\alpha^2}\left(1-e^{-\alpha y}\right) - \frac{R^2\left(R-\alpha^2\beta\right)}{2\alpha^2}c_1 y e^{-\alpha y} \quad \text{...................... (34)}$$

and the velocity near the top is

$$u_t = \frac{c_1}{\alpha^2} + \left(u_0 - \frac{c_1}{\alpha^2}\right)e^{-\alpha(h-y)} + \frac{R^2\left(R-\alpha^2\beta\right)}{2}\left[\left(u_0 - \frac{c_1}{\alpha^2}\right)(y-h)e^{-\alpha(h-y)}\right] \quad \text{... (35)}$$

From the equations (34) and (35) it is noticed that the porosity effects are predominant within the layer given of thickness of the $O(1/\alpha) = O(\sqrt{k})$ from either of the boundaries, beyond which a plug flow (flow with a constant velocity $= \frac{c_1}{\alpha^2} = \frac{c_1 k}{a^2}$ proportional to the coefficient of porosity) is observed.

Then the flow rate

$$q = \int_0^h u\,dy = \frac{c_1}{\alpha^2}\left(h - \frac{2}{\alpha}\right) + \frac{u_0}{\alpha} - \frac{u_0 R^2\left(R-\alpha^2\beta\right)}{2\alpha^2} \quad \text{..........(36)}$$

Hence the mean velocity

$$\bar{u} = \frac{q}{h} = \frac{c_1}{\alpha^2 h}\left(h - \frac{2}{\alpha}\right) + \frac{u_0}{\alpha h} - \frac{u_0 R^2\left(R-\alpha^2\beta\right)}{2\alpha^2 h} \quad \text{.......(37)}$$

Drag force i.e. Shear Stress on the bottom plate for large 'h'

$$\left[\frac{du}{dy} + \beta R\frac{d^2u}{dy^2}\right]_{y=0} = c_1\left[\frac{1}{\alpha^2} - \beta R\right] + \frac{R^2\left(R-\alpha^2\beta\right)c_1}{2\alpha}\left[2\beta R - \frac{1}{\alpha}\right] \quad \text{.............(38)}$$

The Drag force/Shear Stress on the top plate is

$$\left[\frac{du}{dy} + \beta R\frac{d^2u}{dy^2}\right]_{y=h} = \alpha\left(u_0 - \frac{c1}{\alpha^2}\right)\left[1+\alpha\beta R\right] + \frac{R^2\left(R-\alpha^2\beta\right)}{2}\left(u_0 - \frac{c_1}{\alpha^2}\right)\left[1+2\alpha\beta R\right] \quad \text{...(39)}$$

Flow in a Narrow channel i.e. for small 'h' :

Neglecting terms O(h^4) onwards, we get $\sinh \alpha h = \alpha h + \frac{\alpha^3 h^3}{6} + ----$

and $\cosh \alpha h = 1 + \frac{\alpha^2 h^2}{2} +$ - ---- -- -

using these results we get
the net velocity up to the order of R is

$$u = u^{(0)} + Ru^{(1)}$$

$$u = \frac{c_1}{\alpha^2}\left[\frac{1}{h}(y+\frac{\alpha^2 y^3}{6}-\frac{\sinh\alpha y}{\alpha})-\frac{\alpha^2 y^2}{2}+h(\frac{\alpha^2 y}{3}-\frac{\alpha^4 y^3}{36}+\frac{\alpha\sinh\alpha y}{6})+\frac{h^2}{12}(\alpha^4 y^2)\right]$$

$$-\frac{c_1 R}{\alpha^2}(R-\alpha^2\beta)\left[(-\frac{1}{\alpha}+\frac{h^2\alpha}{6})\sinh\alpha y + y\left\{\begin{array}{l}\frac{1}{h}(\frac{\sinh\alpha y}{\alpha}-y-\frac{\alpha^2 y^3}{6})+(1+\frac{\alpha^2 y^2}{2})\\ -h(\frac{\alpha\sinh\alpha y}{6}+\frac{\alpha^2 y}{3}-\frac{\alpha^4 y^3}{36})-\\ \frac{h^2}{12}(\alpha^4 y^2)\end{array}\right\}\right]$$

$$+\ u_0\left[1+(y-h)\frac{R(R-\alpha^2\beta)}{2}\right]\left[\left(\frac{1}{\alpha h}-\frac{\alpha h}{6}\right)\sinh\alpha y\right]$$

…….(40)

the flow rate :

$$q=\int_0^h u\,dy = -\frac{c_1 h^2 R(R-\alpha^2\beta)}{3\alpha^2}+\frac{u_0 h}{2}\left[1-\frac{R\left(R-\alpha^2\beta\right)h\ \alpha^2}{6}\right] \quad \text{……………..(41)}$$

and the mean velocity

$$\bar{u}=\frac{u_0}{2}-\frac{c_1 h\ R(R-\alpha^2\beta)}{3\alpha^2}+\frac{u_0}{2}\left[1-\frac{R\left(R-\alpha^2\beta\right)h\ \alpha^2}{6}\right] \quad \text{…………….(42)}$$

Drag force i.e. shear stress for small h on the bottom on y=o:

$$\left[\frac{du}{dy}+\beta R\frac{d^2u}{dy^2}\right]_{y=0} = c_1\left[\frac{1}{2}+\beta R(-1+\frac{h^2\alpha^2}{6})\right]-\frac{c_1 R(R-\alpha^2\beta)}{\alpha^2}\left[-1+h\alpha^2(\frac{h}{6}-\beta R)\right]$$

$$+u_0(\frac{1}{h}-\frac{\alpha^2 h}{6})\left[1-R(R-\alpha^2\beta)\left\{\frac{h}{2}-\beta R\right\}\right] \quad \text{...(43)}$$

Drag force i.e. shear stress for small h on the Top on y=h:

$$\left[\frac{du}{dy}+\beta R\frac{d^2u}{dy^2}\right]_{y=h} = \frac{c_1 h}{2}\left[-1+\frac{\beta Rh}{3}(1+\alpha^2)\right]-\frac{c_1 R(R-\alpha^2\beta)h}{\alpha}\left[h\alpha+\beta R(-1+\frac{8\alpha}{3})\right]$$

$$+u_0\left[\left\{1+\frac{h^2\alpha^2}{3}+\frac{R(R-\alpha^2\beta)}{2}\right\}+\beta R\left\{\alpha^2+R(R-\alpha^2\beta)(\frac{1}{h}+\frac{h\alpha^2}{3}\right\}\right] \quad (44)$$

Flow for Large porosity i.e. small α

Retaining terms up to α^2 we get

The net velocity upto the first order of R is

$$u = u^{(0)} + Ru^{(1)}$$
$$= c_1\left[\left(hy - y^2\right) - \frac{\alpha^2}{12}\left(h^3 y - 2hy^3 + y^4\right)\right] + \frac{u_0}{h}\left[y + \frac{\alpha^2}{6}\left(y^3 - h^2 y\right)\right] +$$
$$\frac{c_1 R\left(R - \alpha^2\beta\right)}{12}\left[\left(-h^2 y + 3hy^2 - 2y^3\right) + \frac{\alpha^2}{60}\left(7h^4 y - 15h^3 y^2 + 10h^2 y^3 + 30hy^4 - 12y^5\right)\right] +$$
$$\frac{R\left(R - \alpha^2\beta\right)u_0}{2h}\left[\left(y^2 - hy\right) + \frac{\alpha^2}{6}\left(h^3 y - h^2 y^2 - hy^3 + y^4\right)\right]$$
………..(45)

Then the flow rate:

$$q = c_1\left[\frac{h^3}{6} - \frac{\alpha^2 h^5}{60}\right] + u_0\left[\frac{h}{2} - \alpha^2\frac{h^3}{24}\right] + \frac{Rc_1\left(R - \alpha^2\beta\right)}{144}\left(\alpha^2 h^6\right)$$
$$+ \frac{R\left(R - \alpha^2\beta\right)u_0}{120}\left[-10h^2 + 7h^4\alpha^2\right]$$
……….(46)

and mean velocity:

$$\bar{u} = \frac{q}{h} = c_1\left[\frac{h^2}{6} - \frac{\alpha^2 h^4}{60}\right] + u_0\left[\frac{h}{2} - \alpha^2\frac{h^2}{24}\right] + \frac{Rc_1\left(R - \alpha^2\beta\right)}{144}\left(\alpha^2 h^5\right) +$$
$$\frac{R\left(R - \alpha^2\beta\right)u_0}{120}\left[-10h + 7h^3\alpha^2\right]$$
……(47)

The Drag force i.e shear stress for large porosity on the bottom:

$$\left[\frac{du}{dy} + \beta R\frac{d^2u}{dy^2}\right]_{y=0} = c_1\left[\left(h - 2\beta R\right) - \alpha^2\left(\frac{h^3}{12}\right)\right] + u_0\left[\frac{1}{h} - \alpha^2\left(\frac{h}{6}\right)\right] +$$
$$\frac{Rc_1\left(R - \alpha^2\beta\right)}{12}\left[\begin{array}{l}\left(-h^2 + 6h\beta R\right) + \alpha^2\left(\frac{7h^4}{60} - \frac{h^3}{2}\beta R\right) + \\ R\left(R - \alpha^2\beta\right)u_0\left[\left(-\frac{1}{2} + \frac{\beta R}{h}\right) + \alpha^2\left(\frac{h^2}{12} - \frac{h\beta R}{6}\right)\right]\end{array}\right]$$
……….(48)

The Drag force i.e. Shear Stress for large porosity on the top:

$$\left[\frac{du}{dy} + \beta R\frac{d^2u}{dy^2}\right]_{y=h} = c_1\left[-h - 2\beta R + \frac{\alpha^2 h^3}{12}\right] + u_0\left[\frac{1}{h} + \alpha^2\left(\frac{h}{3} + \beta R\right)\right] +$$
$$\frac{Rc_1\left(R - \alpha^2\beta\right)}{12}\left[\left(-h^2 - 6h\beta R\right) + \alpha^2\left(\frac{67h^4}{60} + \frac{5h^3}{2}\beta R\right)\right] + R\left(R - \alpha^2\beta\right)u_0\left[\left(\frac{1}{2} + \frac{\beta R}{h}\right) + \alpha^2\left(\frac{h^2\beta R}{12}\right)\right]$$
……..(49)

Flow for small porosity i.e. large α

We have $\sinh \alpha h = \frac{e^{\alpha h} - e^{-\alpha h}}{2}$ taking $e^{-\alpha h} = 0$ we have $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$

and similarly $\cosh\alpha h \approx \dfrac{e^{\alpha h}}{2}$

we get the net velocity up to the first order of R is given by $u = u^{(0)} + Ru^{(1)}$

$$= \frac{c_1}{\alpha^2} \qquad \text{……….(50)}$$

Hence the flow rate $q = \int_0^h u dy = \dfrac{c_1 h}{\alpha^2}$ ……………(51)

and mean velocity $= \dfrac{c_1}{\alpha^2}$ ……………..(52)

Hence the Drag force/Shear Stress at the bottom $= \left[\dfrac{du}{dy} + \beta R \dfrac{d^2u}{dy^2}\right]_{y=0} = 0$ ….(53)

& the Drag force/Shear Stress at the top $= \left[\dfrac{du}{dy} + \beta R \dfrac{d^2u}{dy^2}\right]_{y=h} = 0$ …………..(54)

Results and Discussions:

Figure 2 illustrates the velocity profile of $u^{(0)}$ for different values of α, and for c_1=1, u_0=1.It is noticed that the velocity curve is almost linear for α=1.37 , convex for α>1.37 and concave for α<1.37

Figure 3 illustrates variations of the velocity $u^{(1)}$ for different values of α, and for β=1, R=0.2, c_1=1, u_0=1. It is observed that the velocity increases with increase in α.

Figure 4 illustrates the net velocity profile of u for different values of α, with β=1 and R=0.2, c_1=1 and u_0=1. it is noticed that the velocity curve is almost linear for α=1.37 , convex for α>1.37 and concave for α<1.37 and velocity decreases with increase in α.

Fig. 5 illustrates the velocities $u^{(0)}$,$u^{(1)}$ and the net velocity u at α=1.37, β=1, R=0.2, c_1=1, h=1 and u_0=1.

Fig. 6 illustrates the net mean velocity u, it is seen that the mean velocity decreases as α increases.

Table 1 and Fig. 7 show the drag force distribution on the top for varying β & R and for α=1, h=1, c_1=1 and u_0=1. It is noticed that the shear stress decreases with increase in β and R.

Table 2 and Fig. 8 show the shear stress/drag force distribution on the bottom for varying β & R and for α=1, h=1, c_1=1 and u_0=1. It is noticed that the shear stress decreases with increase in β, and also decreases with increasing R.

Fig. 9 illustrates the velocity profile of u for different values of α , with β=5, R=0.1, c_1=1, u_0=1 and Large-h=60. The plug flow is noticed and found that the velocity decreases with increase in α.

Fig. 10 shows the drag force/shear stress distribution on the top for varying β, R and α=2, c_1=1, Large-h=50, u_0=1. It is noticed that the shear stress increases with increase in β and up to R=0.2 and then decreases for higher values of R along increasing β.

Fig. 11 shows the drag force/shear stress distribution on the bottom for varying β, R and α =2, c_1=1, Large-h=50, u_0=1. It is noticed that the shear stress decreases with increase in β and R.

Figure 12 illustrates the velocity profile of u for different values of α, with β =5, R=0.5, c_1=1, u_0=1 and small-h=0.1. It is observed that the velocities increase with increase in α.

Fig. 13 and 14 show the drag force/shear stress distribution on the top and bottom for varying β & R and α =2, c_1=1, u_0=1 and Small-h=0.1. It is observed that the shear stress decreases with increase in β and R.

Fig. 15 illustrates the velocity profile of u for different values of small-α, and for R=0.5, β=8, c_1=1, h=1 and u_0=1. Velocities u show low variations for different small-α and decrease with increase in α.

Figures 16 and 17 show drag force/shear stress variations on the top and bottom for varying β & R for small-α =0.1, c_1=1, h=1 and u_0=1. It is noticed that the shear stress decreases with increase in β and R.

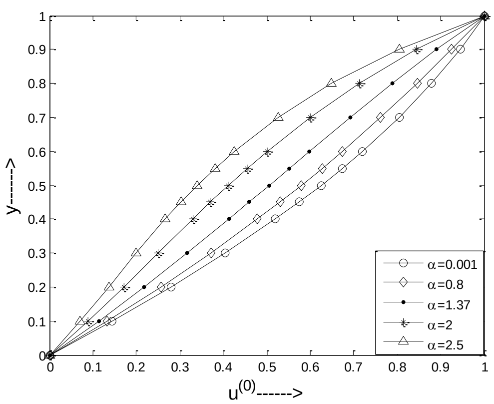

Fig.2 Velocity profile of $u^{(0)}$ for c_1=1, h=1 and u_0=1

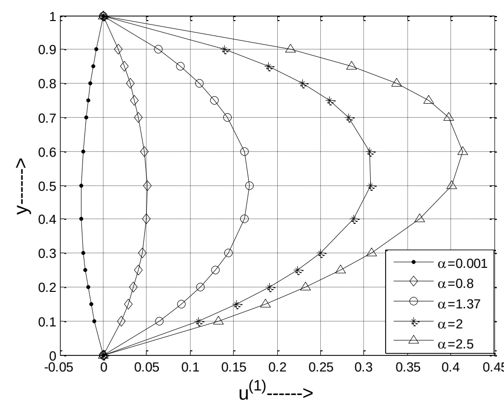

Fig.3 Velocity profile of $u^{(1)}$ for R=0.2, β=1, c_1=1, h=1 and u_0=1

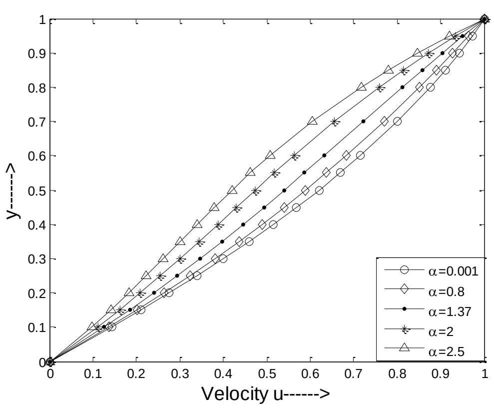

Fig.4 Velocity profile for u for R=0.2, β=1, c_1=1, h=1 and u_0=1

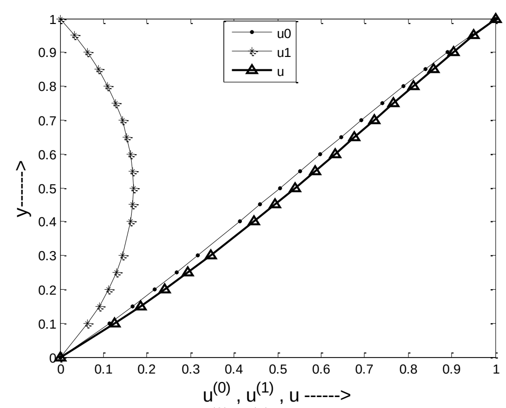

Fig.5 Velocity $u^{(0)}$,$u^{(1)}$ and u for R=0.2, β=1, α=1.37, c_1=1, h=1 and u_0=1

International Journal of Mathematical Sciences, Technology and Humanities 13 (2011) 125 – 139

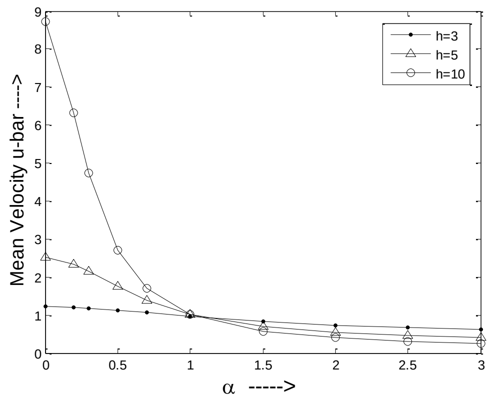

Fig.6 Mean velocity profile of u for $R=0.5$, $\beta=10$, $c_1=1$ and $u_0=1$

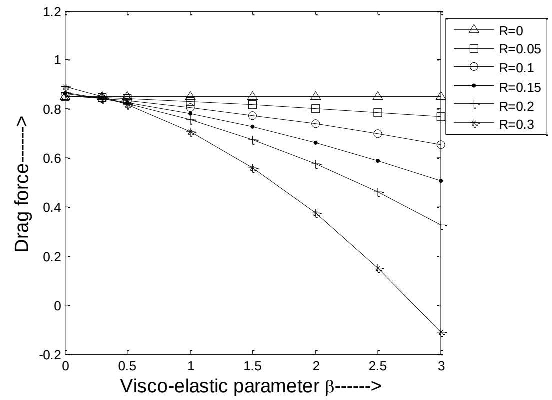

Fig.7 Drag force on the top for $\alpha=1, c_1=1$, $h=1$ and $u_0=1$

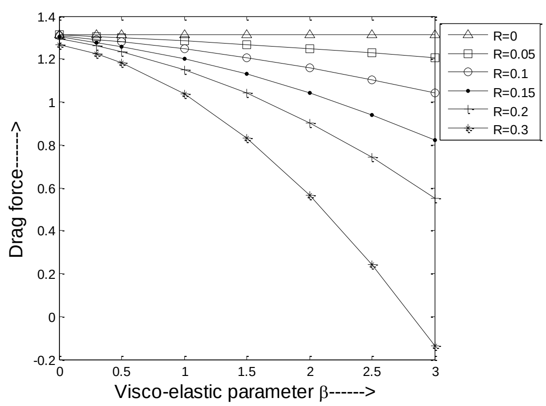

Fig.8 Drag force on the bottom for $\alpha=1$, $c_1=1$, $h=1$ and $u_0=1$

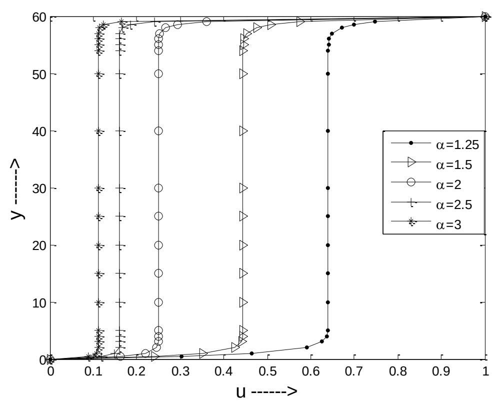

Fig.9 Velocity u for $R=0.1$, $\beta=5, c_1=1$ large-h=60 and $u_0=1$

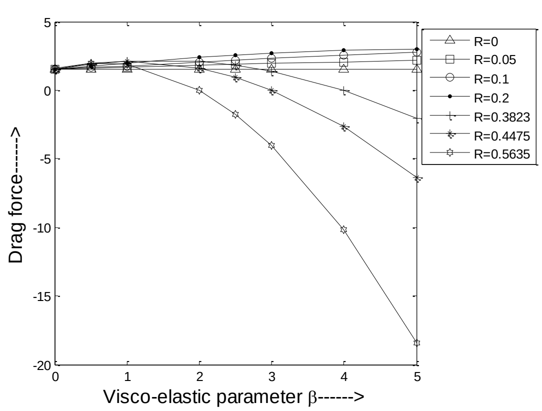

Fig.10 Drag force on the top for $\alpha=2$, $c_1=1$ large-h=50 and $u_0=1$

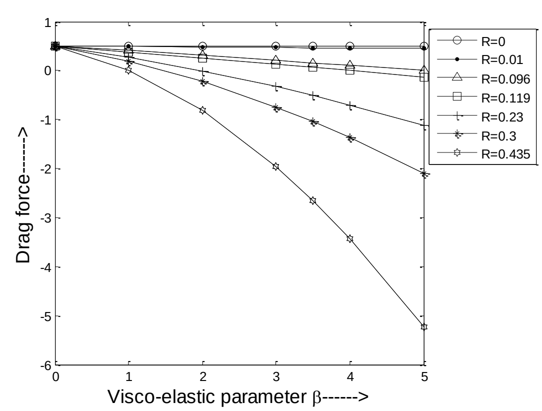

Fig.11 Drag force on the bottom for $\alpha=2$, $c_1=1$, large-h=50 and $u_0=1$

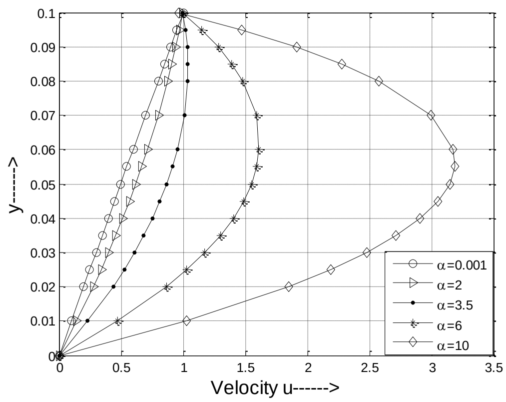

Fig.12 Velocity u for R=0.5, β =5, c_1=1 small-h=0.1 and u_0=1

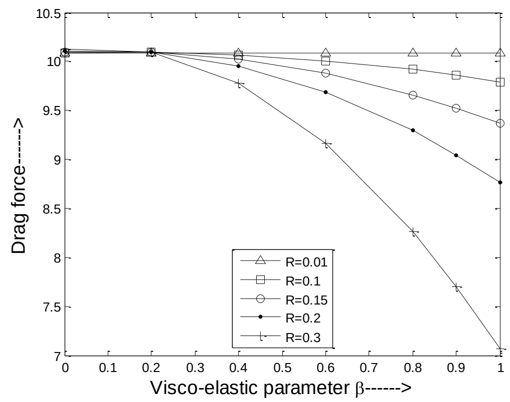

Fig.13 Drag force on the top for α =2, c_1=1 small-h=0.1 and u_0=1

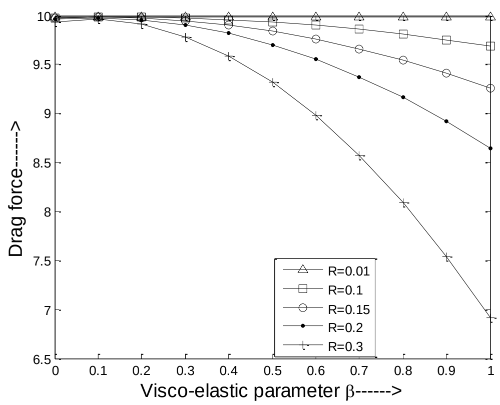

Fig.14 Drag force on the bottom for α =2, c_1=1 small-h=0.1 and u_0=1

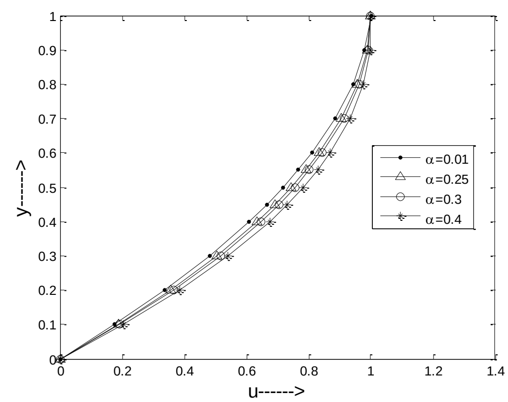

Fig.15 Velocity u for R=0.5, β =8, c_1=1, h=1 and u_0=1 with small- α

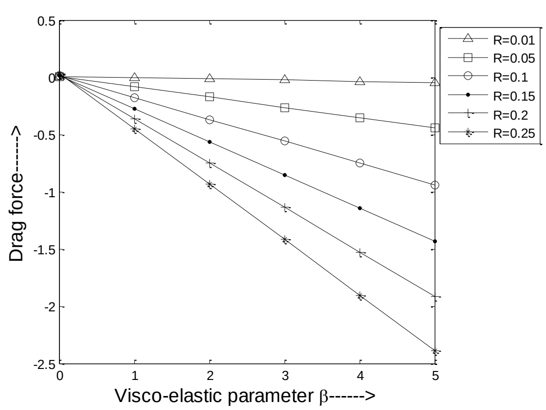

Fig.16 Drag force on the top for small- α=0.1,c_1= 1, h=1 and u_0=1

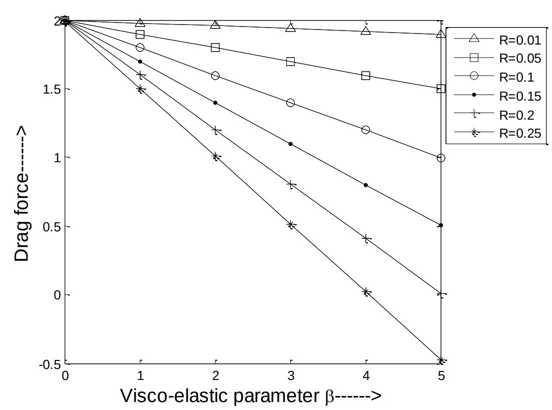

Fig.17 Drag force on the bottom for small- α=0.1, c_1=1, h=1 and u_0=1

International Journal of Mathematical Sciences, Technology and Humanities 13 (2011) 125 – 139
Mohammed Ameenuddin and N. Ch. Pattabhi Ramacharyulu

Table 1 Drag Force /Shear Stress on the Top (Suction Wall) for α=1, h=1 c_1=1 and u_0=1

R \ β	0	0.10	0.20	0.50	1.00	1.50	2.00
0	0.85	0.85	0.85	0.85	0.85	0.85	0.85
0.001	0.85	0.85	0.85	0.85	0.85	0.85	0.85
0.01	0.85	0.85	0.85	0.85	0.85	0.84	0.84
0.03	0.85	0.85	0.85	0.84	0.84	0.83	0.82
0.04	0.85	0.85	0.85	0.84	0.83	0.82	0.81
0.05	0.85	0.85	0.85	0.84	0.83	0.82	0.80
0.06	0.85	0.85	0.85	0.84	0.82	0.81	0.79
0.07	0.85	0.85	0.85	0.84	0.82	0.80	0.78
0.08	0.85	0.85	0.85	0.84	0.81	0.79	0.76
0.09	0.85	0.85	0.85	0.83	0.81	0.78	0.75
0.10	0.86	0.85	0.85	0.83	0.80	0.77	0.74
0.15	0.86	0.85	0.85	0.83	0.78	0.73	0.66
0.18	0.86	0.86	0.85	0.82	0.77	0.70	0.61
0.20	0.87	0.86	0.85	0.82	0.76	0.67	0.58
0.30	0.89	0.88	0.87	0.82	0.71	0.56	0.37
0.40	0.92	0.91	0.89	0.83	0.67	0.44	0.14
0.456	0.94	0.93	0.91	0.84	0.65	0.37	**0.00**
0.60	1.00	0.99	0.98	0.89	0.63	0.21	-0.36

Table 2 Drag Force/Shear Stress on the Bottom (Injection Wall) for α=1, h=1 c_1=1 and u_0=1

R \ β	0	0.1	0.2	0.5	1	1.5	2
0	1.31	1.31	1.31	1.31	1.31	1.31	1.31
0.001	1.31	1.31	1.31	1.31	1.31	1.31	1.31
0.01	1.31	1.31	1.31	1.31	1.31	1.31	1.30
0.03	1.31	1.31	1.31	1.30	1.30	1.29	1.28
0.04	1.31	1.31	1.31	1.30	1.29	1.28	1.26
0.05	1.31	1.31	1.31	1.30	1.28	1.27	1.25
0.06	1.31	1.31	1.31	1.30	1.28	1.26	1.23
0.07	1.31	1.31	1.30	1.29	1.27	1.24	1.22
0.08	1.31	1.31	1.30	1.29	1.26	1.23	1.20
0.09	1.31	1.30	1.30	1.28	1.25	1.22	1.18
0.10	1.31	1.30	1.30	1.28	1.25	1.21	1.16
0.15	1.30	1.29	1.29	1.26	1.20	1.13	1.04
0.18	1.30	1.29	1.28	1.25	1.17	1.08	0.96
0.20	1.29	1.28	1.27	1.24	1.15	1.04	0.90
0.30	1.27	1.26	1.24	1.18	1.04	0.83	0.57
0.437	1.22	1.20	1.19	1.10	0.86	0.49	**0.00**
0.50	1.19	1.18	1.16	1.06	0.77	0.32	-0.30
0.615	1.12	1.12	1.10	1.00	0.63	**0.00**	-0.87
0.70	1.07	1.07	1.06	0.96	0.53	-0.23	-1.30

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