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STEADY FLOW OF A VISCOUS FLUID THROUGH A SATURATED POROUS MEDIUM OF A FINITE THICKNESS, THE BOTTOM OF WHICH IS IMPERMEABLE WITH A VELOCITY SLIP AND THERMALLY INSULATED WHILE THE OTHER SIDE IS STRESS FREE AND KEPT AT A CONSTANT TEMPERATURE WITH A UNIFORMLY DISTRIBUTED CONSTANT HEAT SOURCE.

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Abstract:
This paper deals with a steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable and thermally insulated bottom with a velocity slip and with a uniformly distributed constant heat source in the flow region. Exact solutions of Momentum and Energy equations are obtained when the temperatures on the fixed bottom and on the free surface are prescribed. Flow rate , Mean velocity , Temperature , Mean Temperature , Mean Mixed Temperature in the flow region and the Nusselt number on the free surface have been obtained. The cases of small values of porosity coefficient and large depth (deep fluid) are also discussed. The results are illustrated graphically.

Key Words: Porous Medium , Velocity , Temperature , Porosity Coefficient, slip velocity, heat source.

Introduction:

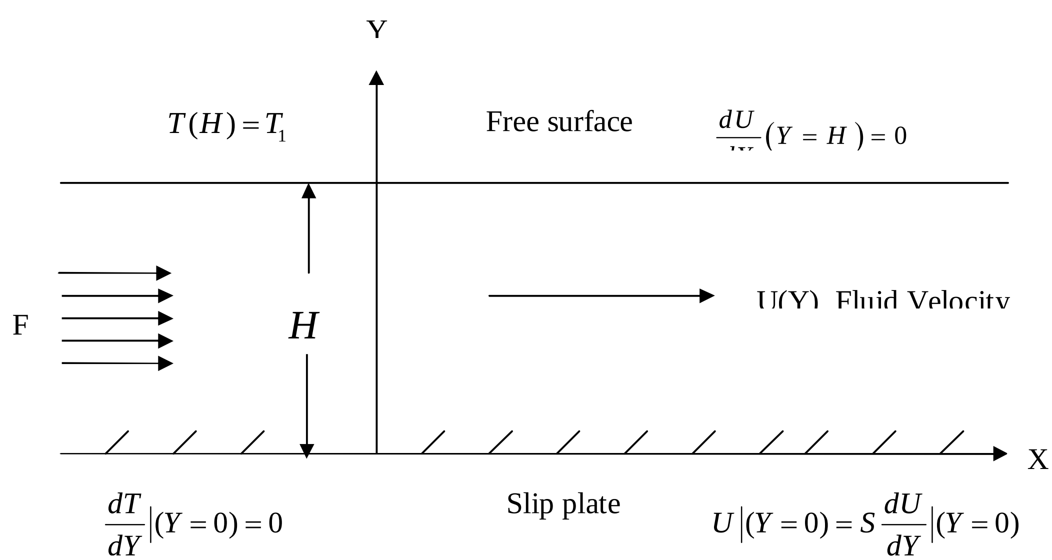
Bevers and Joseph[1] considered in addition to the classical Darcy's law, a slip boundary condition at the interface between the porous medium and a clear medium, the slip velocity for the free flow is proportional to the shear rate at the permeable boundary. Subsequently, Saffman[2] provided a theoretical justification for the Bevers and Joseph[1] boundary condition. Taylor[3], and Richardson[4] carried out exact analysis of the situation assuming a different model together with the Bever's and Joseph[1] interface condition. Nagaratnam [5] studied some problems in thermo-viscous fluid dynamics.Rajesh Yadav [6] studied convective heat transfer through a porous medium in channels and pipes. Khaja Moinuddin and Pattabhi Ramacharyulu [7] studied steady flow of a viscous fluid through a saturated porous medium of a finite thickness, the bottom of which is impermeable with a velocity slip and thermally insulated while the other side is stress free and kept at a constant temperature.

In this paper the steady flow of a viscous fluid of viscosity μ and of finite depth H through a porous medium of permeability coefficient 'k*' over a fixed impermeable , thermally insulated bottom with a velocity slip 'S' and with a constant heat source 'F' distributed uniformly in the flow region is investigated. The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized darcy's law proposed by Yama Moto and Iwamura[8] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

The basic equations of momentum and energy are solved to get exact expressions for the velocity and temperature distributions. Employing these , the flow rate , mean velocity, mean temperature, mean mixed temperature and the Nusselt number on the free surface have been obtained and their variations are illustrated graphically.
The cases of (i) Low porosity coefficient (large α) and (ii) Large depths (large h) are also discussed.

Mathematical formulation.
Consider the steady forced convective flow of a viscous fluid of viscosity μ through a saturated porous medium of finite depth H over a fixed horizontal impermeable bottom. The flow is generated by a constant pressure gradient parallel to the plate. Further the bottom is thermally insulated with a velocity slip 'S'. The free surface is exposed to the atmospheric temperature T_1 with a constant heat source F distributed uniformly in the flow region.

With reference to a rectangular Cartesian coordinate system with the origin O on the bottom, the X-axis in the flow direction (i.e. parallel to the applied pressure gradient) and the Y-axis vertically upwards. The bottom is represented as Y=0 and the free surface as Y=H.



Flow Configuration

Basic equations:
Let the convective flow be characterized by the velocity field U = (U(Y), 0,0)and the temperature T(Y). The choice of the velocity satisfies the continuity equation

$$divU = 0 \qquad --(1)$$

The momentum equation:

$$-\frac{\partial P}{\partial X}+\mu\frac{d^2U}{dY^2}-\frac{\mu U}{k^*}=0 \qquad \text{--(2)}$$

and the energy equation :

$$\rho c U\frac{dT}{dX}=K\frac{d^2T}{dY^2}+\mu\left(\frac{dU}{dY}\right)^2+F \qquad \text{--(3)}$$

In the above equations ρ is the fluid density, k^* the coefficient of porosity of the medium, ‘c’ is the specific heat, K the thermal conductivity of the fluid , P the fluid pressure and F a constant heat source distributed uniformly in the flow region.
Boundary conditions:

Since the fluid slips on the bottom $U(0)=S\frac{dU}{dY}\Big|_{Y=0}$ --(4a)

where S is the slip parameter.

At the free surface shear stress= $\mu U^1(H)=0$ --(4b)

The bottom plate is thermally insulated:

$\therefore T^1(0)=0$ --(5a)

The free surface is exposed to the atmosphere

$\therefore T(H)=T_1$ = temperature of the atmosphere. --(5b)

In terms of the non-dimensional variables defined hereunder:

X=ax ; Y=ay ; H=ah ; $U=\frac{\mu u}{\rho a}$; $P=\frac{\mu^2 p}{\rho a^2}$; $k^*=\frac{a^2}{\alpha^2}$, $-\frac{\partial P}{\partial X}=\frac{\mu^2}{\rho a^3}c_1,\left(c_1=\frac{-\partial p}{\partial x}\right)$;

$T=T_0+(T_1-T_0)\theta$, $\frac{\partial T}{\partial X}=\frac{(T_1-T_0)c_2}{a}$ where $c_2=\frac{\partial\theta}{\partial x}$ P_r (Prandtl number)= $\frac{\mu c}{K}$;

$E=\frac{\mu^3}{\rho^2a^2K(T_1-T_0)}$, s=S/a , $f=\frac{a^2F}{K(T_1-T_0)}$ --(6)

(where a is some standard length and T_0 the temperature at the bottom) the basic field equations can be rewritten as :
Momentum equation:

$$\frac{d^2u}{dy^2}-\alpha^2u=-c_1 \qquad \text{--(7)}$$

and energy equation:

$$\frac{d^2\theta}{dy^2}=P_rc_2u-E\left(\frac{du}{dy}\right)^2-f \qquad \text{--(8)}$$

together with the boundary conditions for velocity

$u(0)=s\frac{du}{dy}\Big|_{y=0}$ and $\frac{du}{dy}\Big|_{y=h}=0$ --(9)

and for temperature

$$\frac{d\theta}{dy}\Big|_{y=0} = 0 \quad \text{and} \quad \theta(h) = 1 \qquad \text{--(10)}$$

The solution of these equations together with the related boundary conditions yield the following results.

The velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2}\left[1 - \frac{\cosh \alpha(h-y)}{(\cosh(\alpha h) + s\alpha \sinh(\alpha h))}\right] \qquad \text{--(11)}$$

The flow rate in the non-dimensional form is

$$q = \int_0^h u(y)dy = \frac{c_1}{\alpha^2}\left(h - \frac{\sinh(\alpha h)}{\alpha(\cosh(\alpha h) + s\alpha \sinh(\alpha h))}\right) \qquad \text{--(12)}$$

Mean velocity $\bar{u} = \frac{1}{h}\int_0^h u(y)dy = \frac{c_1}{h\alpha^2}\left[h - \frac{\sinh \alpha h}{\alpha(\cosh(\alpha h) + s\alpha \sinh(\alpha h))}\right]$ --(13)

The temperature distribution:

$$\theta(y) = 1 + \frac{P_r c_1 c_2}{\alpha^2}\left\{\frac{(y^2 - h^2)}{2} + \frac{(h-y)\sinh \alpha h}{\alpha^3(\cosh(\alpha h) + s\alpha \sinh(\alpha h))} + \frac{(1 - \cosh \alpha(h-y))}{\alpha^2(\cosh(\alpha h) + s\alpha \sinh(\alpha h))}\right\} -$$

$$\frac{Ec_1^2}{2\alpha^2(\cosh(\alpha h) + s\alpha \sinh(\alpha h))^2}\left\{\frac{(h^2 - y^2)}{2} + \frac{(y-h)\sinh 2\alpha h}{2\alpha} + \frac{(\cosh 2\alpha(h-y) - 1)}{4\alpha^2}\right\} + \frac{f(h^2 - y^2)}{2}$$

--(14)

Further the mean temperature in the non-dimensional form is given by

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta(y)dy$$

$$=\frac{1}{h}\left\{\begin{array}{l} h+\frac{P_r c_1 c_2}{\alpha^2}\left(\begin{array}{l}\frac{-h^3}{3}+\frac{h^2\sinh\alpha h}{2\alpha^3\left(\cosh(\alpha h)+s\alpha\sinh(\alpha h)\right)}-\frac{h}{\alpha^2\left(\cosh(\alpha h)+s\alpha\sinh(\alpha h)\right)}-\\ \frac{\sinh(\alpha h)}{\alpha^3\left(\cosh(\alpha h)+s\alpha\sinh(\alpha h)\right)}\end{array}\right)- \\ \frac{Ec_1^2}{2\alpha^2\left(\cosh(\alpha h)+s\alpha\sinh(\alpha h)\right)^2}\left(\frac{h^3}{3}-\frac{h^2\sinh 2\alpha h}{4\alpha}-\frac{h^2}{4\alpha^2}+\frac{\sinh 2\alpha h}{8\alpha^3}\right)\end{array}\right\}+\frac{fh^2}{3} \quad \text{--(15)}$$

and the mean mixed temperature in the dimensionless form is

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy}=\frac{1}{\left(h-\frac{\sinh\alpha h}{\alpha D}\right)}\left\{\begin{array}{l}\left(h-\frac{\sinh\alpha h}{\alpha D}\right)+\frac{P_r c_1 c_2}{\alpha^2}\left(\begin{array}{l}\frac{-h^3}{3}+\frac{h}{\alpha^2 D}-3\frac{\sinh\alpha h}{\alpha^3 D}+3\frac{h^2\sinh\alpha h}{2\alpha D}\\ +\frac{2h}{\alpha^2 D}+\frac{h}{\alpha^2 D^2}+\frac{\sinh 2\alpha h}{4\alpha^3 D^2}-2\frac{\sinh\alpha h}{\alpha^3 D^2}\\ -\frac{h\sinh^2\alpha h}{\alpha^2 D^2}+\frac{\sinh(\alpha h)\cosh(\alpha h)}{\alpha^3 D^2}\end{array}\right)- \\ \frac{Ec_1^2}{2\alpha^2(D)^2}\left(\begin{array}{l}\frac{h^3}{3}-\frac{h}{4\alpha^2}+\frac{\sinh 2\alpha h}{8\alpha^3}-\frac{h^2\sinh 2\alpha h}{4\alpha}+\frac{h}{\alpha^2 D}-\\ \frac{\sinh\alpha h}{\alpha^3 D}+\frac{h^2\sinh\alpha h}{2\alpha D}-\frac{\sinh 2\alpha h}{2\alpha^3 D}+\frac{1}{12\alpha^3 D}-\\ \frac{h\sinh(\alpha h)\sinh 2(\alpha h)}{2\alpha^2 D}+\frac{\sinh 2(\alpha h)\cosh(\alpha h)}{2\alpha^3 D}\end{array}\right)+ \\ \frac{f}{D}\left(\frac{h^3}{3}-\frac{h}{\alpha^2}+\frac{\sinh\alpha h}{\alpha^3}-\frac{h^2\sinh\alpha h}{2\alpha}\right)\end{array}\right\} \quad \text{--(16)}$$

where D= $\left(\cosh(\alpha h)+s\alpha\sinh(\alpha h)\right)$.

Heat transfer coefficient Nusselt number on the free surface:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{P_r c_1 c_2}{\alpha^2}\left(h - \frac{\sinh \alpha h}{\alpha(\cosh(\alpha h) + s\alpha \sinh(\alpha h))}\right) + \frac{Ec_1^2}{2\alpha^2(\cosh(\alpha h) + s\alpha \sinh(\alpha h))^2}\left(h - \frac{\sinh 2\alpha h}{2\alpha}\right) - fh \qquad \text{--(17)}$$

Case.1: For large values of α i.e for low porosity the asymptotic flow characteristics are the following:

For large α $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\tanh \alpha h \approx 1$ and neglecting the terms of $O(\frac{1}{\alpha^3})$ we get

The velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2}\left(1 - \frac{e^{-\alpha y}}{(1+s\alpha)}\right) \qquad \text{--(18)}$$

Flow rate :

$$q = \frac{c_1}{\alpha^2} h \qquad \text{--(19)}$$

Mean velocity:

$$\bar{u} = \frac{c_1}{\alpha^2} \qquad \text{--(20)}$$

Temperature:

$$\theta(y) = 1 + \frac{p_r c_1 c_2 (y^2 - h^2)}{2\alpha^2} + \frac{f(h^2 - y^2)}{2} \qquad \text{--(21)}$$

Mean temperature:

$$\bar{\theta} = 1 - \frac{p_r c_1 c_2}{3\alpha^2} h^2 + \frac{fh^2}{3} \qquad \text{--(22)}$$

Mean mixed temperature:

$$\theta_M = 1 + \frac{p_r c_1 c_2}{\alpha^2}\left(\frac{3h^2}{2} - \frac{\alpha(1+s\alpha)h^3}{3}\right) - \frac{fh^2}{(h\alpha(1+s\alpha)-1)} \qquad \text{--(23)}$$

Heat transfer coefficient Nusselt number on the free surface:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{p_r c_1 c_2}{\alpha^2} h - fh \qquad \text{--(24)}$$

Case.2: Flow for large depths i.e for large h:

For large h $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\tanh \alpha h \approx 1$ and neglecting terms of $O(\frac{1}{h^3})$.

Velocity: $u(y) = \frac{c_1}{\alpha^2}\left(1 - \frac{e^{-\alpha y}}{(1+s\alpha)}\right)$ --(25)

Flow rate :

$$q = \frac{c_1}{\alpha^2}\left(h - \frac{1}{\alpha(1+s\alpha)}\right) \qquad \text{--(26)}$$

Mean velocity:

$$\bar{u} = \frac{c_1}{\alpha^2 h}\left(h - \frac{1}{\alpha(1+s\alpha)}\right) \qquad \text{--(27)}$$

Temperature:

$$\theta(y) = 1 + \frac{p_r c_1 c_2}{\alpha^2}\left\{\frac{(y^2-h^2)}{2} + \frac{h-y}{\alpha^3(1+s\alpha)} - \frac{e^{-\alpha y}}{\alpha^2(1+s\alpha)}\right\} + \frac{Ec_1^2}{\alpha^2(1+s\alpha)^2}\left\{-\frac{e^{-2\alpha y}}{4\alpha^2} + \frac{h-y}{2\alpha}\right\} + \frac{f(h^2-y^2)}{2} \qquad \text{--(28)}$$

Mean temperature:

$$\bar{\theta} = 1 + \frac{p_r c_1 c_2}{\alpha^2}\left(\frac{-h^2}{3} + \frac{h}{2\alpha^3(1+s\alpha)} - \frac{1}{\alpha^3(1+s\alpha)}\right) + \frac{Ec_1^2}{\alpha^2(1+s\alpha)^2}\left(-\frac{1}{8h\alpha^3} + \frac{h}{4\alpha}\right) + \frac{fh^2}{3} \qquad \text{--(29)}$$

Mean mixed temperature:

$$\theta_M = \frac{1}{\left[h - \frac{1}{\alpha(1+s\alpha)}\right]}\left[\begin{array}{l}\left(h - \frac{1}{\alpha(1+s\alpha)}\right) + \frac{p_r c_1 c_2}{\alpha^2}\left(\begin{array}{l}\frac{-h^3}{3} - \frac{3}{\alpha^3(1+s\alpha)} + 3\frac{h^2}{2\alpha(1+s\alpha)} - \\ \frac{h}{\alpha^2(1+s\alpha)^2} + \frac{1}{2\alpha^3(1+s\alpha)^2} \\ + \frac{1}{\alpha^2(1+s\alpha)^2}\end{array}\right) \\ -\frac{Ec_1^2}{2\alpha^2}\left(\frac{1}{4\alpha^3(1+s\alpha)^2} - \frac{h^2}{2\alpha(1+s\alpha)^2} - \frac{h}{\alpha^2(1+s\alpha)^3} + \frac{1}{\alpha^3(1+s\alpha)}\right) \\ + f\left(\frac{1}{\alpha^3(1+s\alpha)} - \frac{h^2}{2\alpha(1+s\alpha)}\right)\end{array}\right] \qquad \text{--(30)}$$

Heat transfer coefficient Nusselt number on the free surface:

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{p_r c_1 c_2}{\alpha^2}\left(h - \frac{1}{\alpha(1+s\alpha)}\right) - \frac{Ec_1^2}{2\alpha^3(1+s\alpha)^2} - fh \qquad \text{--(31)}$$

Results and Discussion:

1.Fig 1 illustrates that the velocity gradually decreases for the increasing values of the porosity parameter α for smaller slip s= 0.2. In the case of the large α velocity of the fluid flow also decreases with the increase in the values of α (Fig 2) .For larger depths the thickness of the boundary layer decreases with the increase in the porosity parameter α (Fig 3).

2.Mean velocity of the fluid appears to be increasing for the increase in the slip parameter s and smaller values of the porosity parameter α (Fig 4). In the case of the large α mean velocity decreases with the increase in the porosity parameter α (Fig 5). For different slip parameters mean velocity remains unaltered when α >0.3 in the case of large h (Fig 6).

3.Temperature of the flow region slightly increases with the increasing values of porosity parameter when f=10 and remains constant for f=100 (Fig.7 , Fig.8). Temperature increases for increasing smaller values of α and remain constant for larger values of α (Fig.9). It is observed that the temperature increases with the increase in porosity parameter α (Fig.10).

4.Fig.11 illustrates that the mean temperature remains constant for the increasing values of the prandtl number 'p' for porosity parameter $\alpha > 0.3$ and decreases for the increasing values of 'p' and $\alpha < 0.3$. In the case of large α it is noticed from Fig.12 that, mean temperature decreases with the increasing prandtl numbers. In the case of large 'h' mean temperature decreases with the increase in slip parameter 's' and almost remain constant for the porosity parameter $\alpha < 0.2$ (Fig.13).

5.Similar behavior of the mean temperature is observed for the mean mixed temperature when α >0.2 and decreases for the increasing 'p' and α <0.2.(Fig.14). In the case of large α mean mixed temperature increases with the increasing slip parameter s (Fig.15).In the case of large 'h' mean mixed temperature decreases with the increase in the slip parameters(Fig.16).

6.The rate of heat transfer increases with the increase in the values of the prandtl number (Fig 17).In the case of large α the heat transfer rate increases with the increasing prandtl number 'p'(Fig.18).Heat transfer rate in the case of large 'h' increases with the increase in the slip parameter 's' and for smaller values of α (Fig.19).

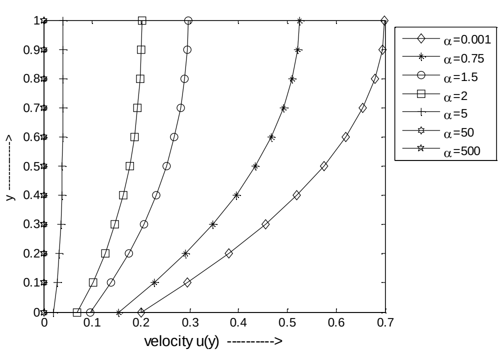


fig.1 velocity profile for s=0.2,h=1,c1=1

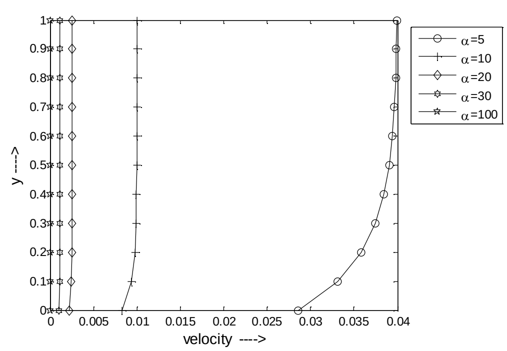


fig.2 velocity Profile for large α &for h=1

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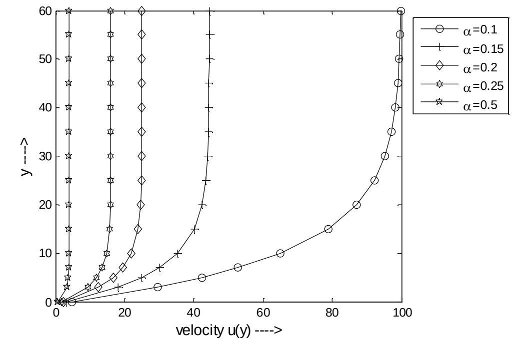

fig.3 velocity Profile for c1=1 and large h=60

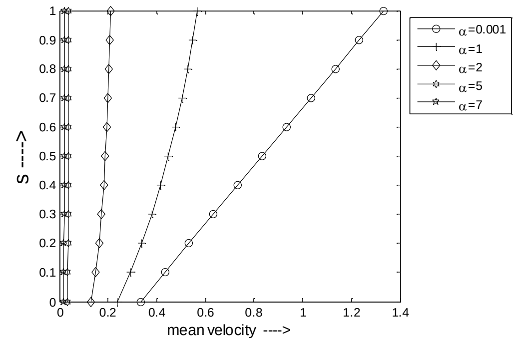

fig.4 mean velocity for h=1

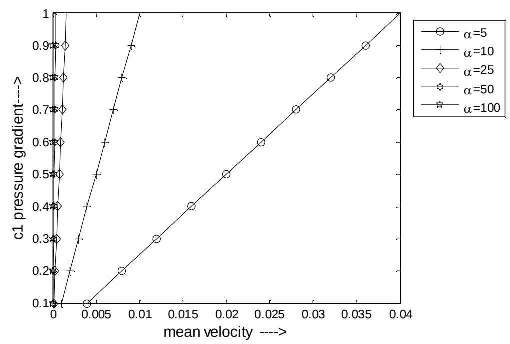

fig.5 mean velocity Profile for large α &for h=1

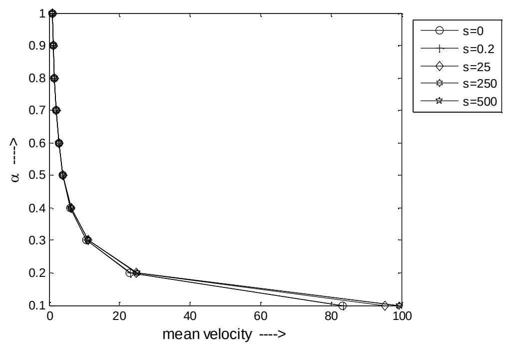

fig.6 mean velocity Profile for large h=60 and c1=1

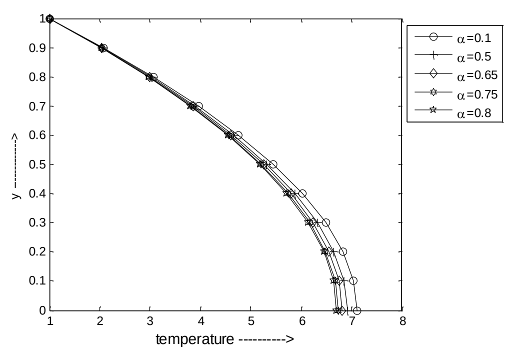

fig.7 Temperature distribution for p=1 ,h=1,E=5,c1=1,c2=1,f=10

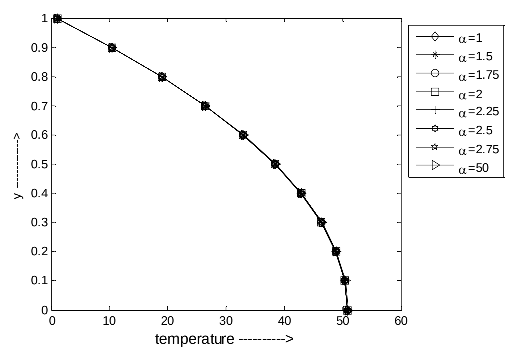

fig.8 Temperature distribution for f=100,p=1 s=.5,h=1,E=1,c1=1,c2=1

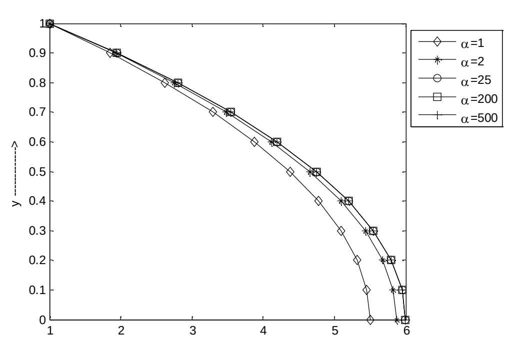

fig.9 Temperature distribution for large α and for f=10, p=1, s=.2,h=1,E=5,c1=1,c2=1

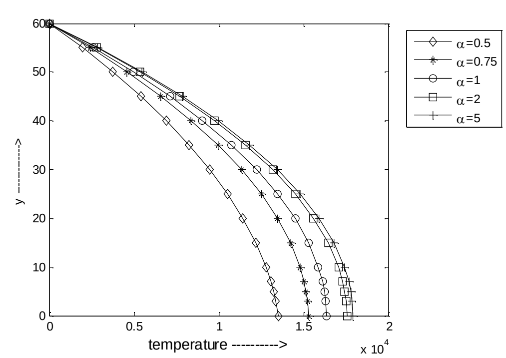

fig.10 Temperature distribution for f=10, p=1 s=.2,h=60,E=5,c1=1,c2=1

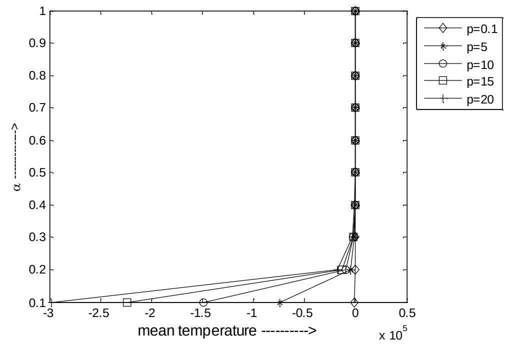

fig.11 mean temperature distribution for,s=.5,f=10,h=1,E=1,c1=1,c2=1

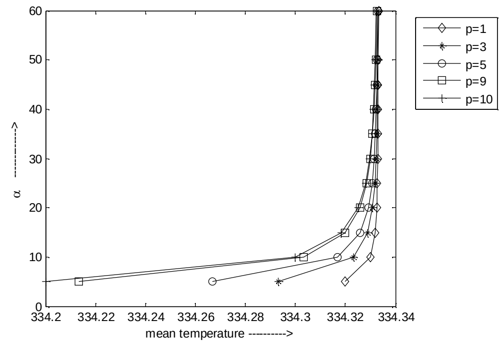

fig.12 Mean temperature for large α and for f=1000 p=1 ,h=1,E=5,c1=1,c2=1

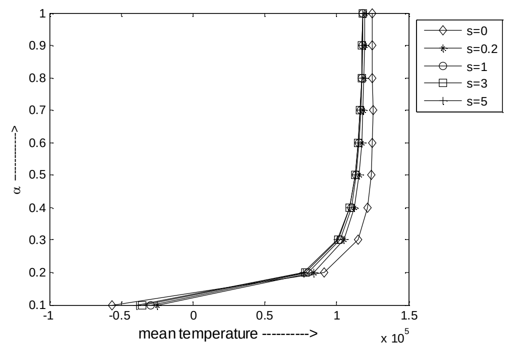

fig.13 mean temperature for f=100, p=1 ,h=60,E=5,c1=1,c2=1

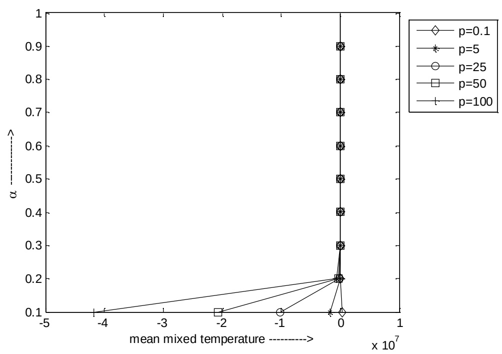

fig.14 mean mixed temperature distribution for f=10,s=.5,h=1,E=1,c1=1,c2=1

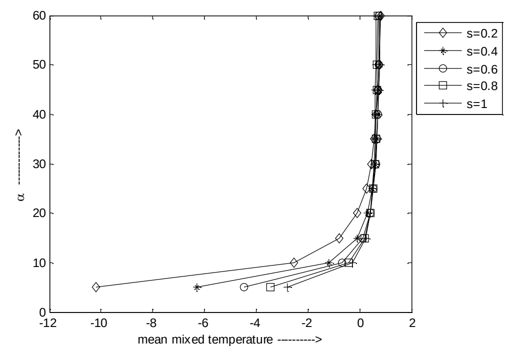

fig.15 Mean mixed temperature for large α and for f=100, p=1 ,h=1,E=5,c1=1,c2=1

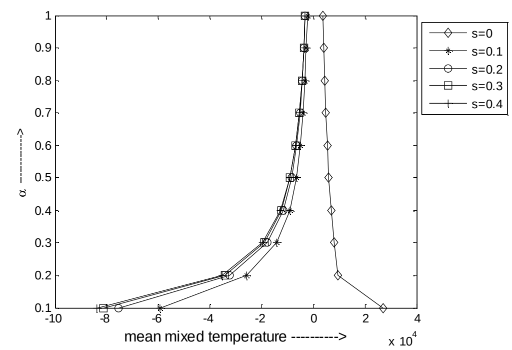

fig.16 mean mixed temperature for f=100, p=1 ,h=60,E=5,c1=1,c2=1

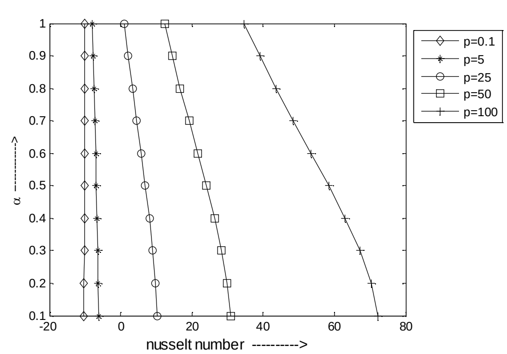

fig.17 nusselt number for f=10,s=.5,h=1,E=1,c1=1,c2=1

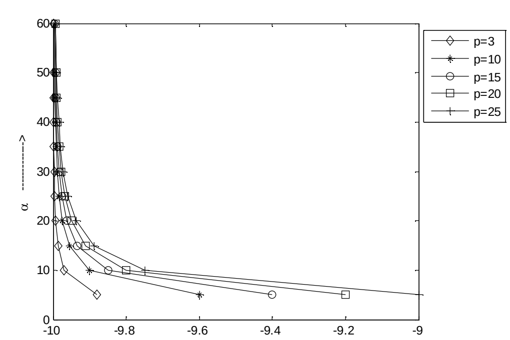

fig.18 Nusselt number on the top for large α and for f=10 ,h=1,c1=1,c2=1

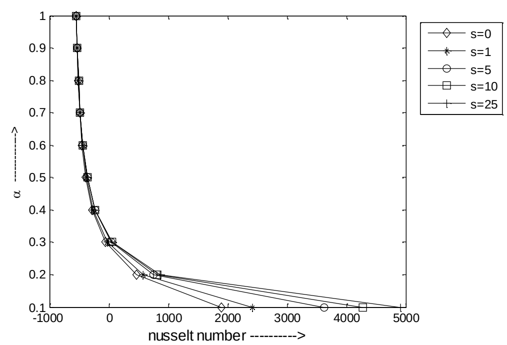

fig.19 nusselt number for large h=60,f=10 p=1 ,E=5,c1=1,c2=1

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