INTERNATIONAL JOURNAL OF MATHEMATICAL SCIENCES, TECHNOLOGY AND HUMANITIES

www.internationalejournals.com

Available online at www.internationaleJournals.com

International eJournals

International Journal of Mathematical Sciences, Technology and Humanities 40 (2012) 398 – 407

Fixed Points of Mappings Satisfying Semi-Contractivity Conditions

B.Nageswara Rao

Department of Basic Science and Humanities (Mathematics), Costal Institute of Technology and Management, Narapam,Kothavalasa-535183, Vijayanagaram district, A.P., India

Abstract: In this paper,we have discussed fixed points of mapping satisfying contractivity conditions.

Key Words: Contractive conditions, duality mapping,contractive mapping

Introduction.

Semi-contractive and semi accer-ative mappings in Banach spaces was discussed by Browder, F.E. (1). Fixed points of Mappings satisfying semi-contractivity conditions. Were discussed by Schu, J.(2). Further information can be had from (3),(4),(5) and (6).

After doing this we have proved theorems regarding the existence of fixed points of asymptotically semi – contractive mappings.

1. PRELIMINARIES:

A 2-normed apace (E, $\| \cdot , \cdot \|$) is called uniformly convex if for each $\varepsilon > O$ there exists a δ (E) > O Such that if x,y,a ε E with $\| x,a \|$, $\| y,a \| <1$ and $\| x - y, a \| \geq \varepsilon$ it follows that $\| x + y, a \| < 2 (1 - \delta (\in))$ Equivalently, E is uniformly convex if whenever $\| X_n , a \|$, $\| Y_n , a \|$ ε E with $\| x_n , a \|$, $\| y_n , a \| \leq 1$ and $\| x_n + y_n , a \| \to 2$, then $\| x_n + y_n , a \| \to O$.

(E, $\| \cdot , \cdot \|$) is (uniformly) Smooth if the 2 - norm of E is (uniformly) Gateaux – differentiable on the boundary of the unit sphere in E. Furthermore, (E, $\| \cdot , \cdot \|$) is said to satisfy opial condition if for each sequence (x_n) ε E^N which converges weakly to some x ε E it follows that

$$\text{Lim inf} \| x_n - x , a \| \leq \text{lim inf} \| x_n - y , a \| , \ \forall \ y \ \varepsilon \ E/_{\{x\}} \text{ and a } \varepsilon \text{ E .}$$

For a given gauge function μ , this means for a mapping $\mu : R^+ \to R^+$

which is continuous and strictly increasing with μ(o) = O and lim μ(x) = ∞, the n$\to\infty$ related set valued duality mapping J_E^μ: E $\to 2^{E^*}$ is given by J_E^μ (x , a) = { u ε E* / u(x , a) = ‖ u ‖ ‖ x , a ‖ and ‖ u ‖ = μ ‖ x , a ‖ } for all x, a ε E.

A mapping J : E $\to$ E* is called duality mapping with respect to μ if J (x , a) ε J_E^μ (x, a) for all x,a ε E. such a mapping J is said to be weakly sequentially continuous if for all x_n ε E^N all (x,a) ε E it follows from $x_n \to$ x that (J(x_n , a)) $\to$ J (x , a) (as usual $\to$ and $\to$stand for week and weak$_*$ convergence respectively, while strong convergence of a sequence respectively, while strong convergence of a sequence z_n to a point z is indicated by lim z_n = z). It is well known that J_E^μ is single valued iff (E, ‖ . , . ‖) is smooth. In this we regard J_E^μ a mapping from E $\to$ E*.

In all our proofs we assume, without loss of generality that μ = I, the identity mapping. For abbreviation we set J_E = J_E^I . Furthermore in the sequel E is always assumed to be a linear space over the real field.

(1.2) Fixed Point of Asymptotically Semi-Contractive And Weakly Asymptotically Semi-Contractive Mappings:

(1.2.1) DEFINITION: Let (E, ‖ . , . ‖) be a 2 – normed space, $\phi \neq$ A $\subset$ E and T : A $\to$ A.

(a) T is called Lipschitzian with constant L $\geq$ O if ‖ Tx – Ty , a ‖ $\leq$ L ‖ x – y , a ‖ $\forall$ x , y ε A .

(b) T is called non-expansive if T is Lipschitzian with constant L = 1.

(c) T is called a Banach contraction if T is Lipschitzian with constant L < 1 .

(d) T is called compact if T is continuous and maps bounded sets into relatively compact ones.

(e) T is strongly compact (or completely continuous) if T is continuous form weak topology of E to the strong topology of E..

(f) T is called asymptotically non-expansive with sequence (k_n) ε $[1,\infty]^N$ if lim (k_n) = 1 and ‖ T^n (x) – T^n (y) , a ‖ $\leq$ k_n ‖ x – y , a ‖ for all n ε N and all x,y,a ε A.

<u>(1.2.2) DEFINITION</u>: Let (E, ‖ . , . ‖) be a 2 – normed space, $\phi \neq$ A $\subset$ E and T : A $\to$ A

(a) T is called asymptotically semi-contractive if there exists a mapping S : A x A x A $\to$ A and a sequence (k_n) ε (l, $\infty)^N$ such that Tx = S(x, x, x) for all x ε A, while for each fixed x, a ε A, S(., x, a) is asymptotically

Non-expansive with sequence (k_n) and S (a, x, .) is strongly compact.

(b) T is called weakly asymptotically semi-contractive if there exists a mapping S : A x A x A $\rightarrow$ A and a sequence (k_n) ε (1 , ∞)^N such that Tx = S (x, x, x) for all x ε A , while for fixed x ε A and fixed n ε N the mapping y $\rightarrow$ S(. , y, a) ^{n} (x) is compact on A.

Remark : Clearly, every semi-contractive mapping is asymptotically too. But the converse is not true. This we conclude from [ ].

(1.2.3) LEMMA : Let (E, ‖ . , . ‖) be a 2 – Banach space, $\phi \neq A \subset E$ closed and λ ε [0 , 1] . Suppose S : A x A x A $\rightarrow$ A is such that
$\| S (y_1 , z_1 , x) – S (y_2 , z_2 , x) , a \| \leq \lambda (\| y_1 – y_2 , a \| + \| z_1 – z_2 , a \|)$ for all x , y_1, y_2 , z_1, z_2 ε A. Then

(a) there is exactly one mapping R : A $\rightarrow$ A sich that S (Rx, x, x) = Rx for all x ε A

(b) $\| Rz – Rw , a \| < \left(\frac{1}{1-\lambda}\right) \| S (Rw , z , z) – S (Rw , w , w) a \|$

Proof : For each x ε A, the mapping S (. , . , x) : A $\rightarrow$ A is a contraction and thus has a unique fixed point Rx in A by analogue contraction fixed point theorem in 2 – Banach space, This establishes (a) . To show (b).fix z,w ε A. Then ‖ Rz – Rw , a ‖ = ‖ S (Rz, z, z) – S (Rw, w, w) , a ‖ ≤ ‖ S(Rz, z, z) – S (Rw , z , z) + S(Rw, z, z) – S(Rw, w, w) , a‖ ≤ ‖ S(Rz, z, z) – S (Rw , z , z), a ‖ + S(Rw, z, z) – S(Rw, w, w) , a‖
≤ ‖ Rz – Rw , a ‖ + ‖ S (Rw, z, z) – S (Rw, w , w) , a ‖

Or , $(1 - \lambda) \| Rz – Rw, a \| \leq \| S (Rw, z, z) – S (Rw, w,w) , a \|$

Or, $\| Rz – Rw,a \| \leq \frac{1}{1-\lambda} \| S (Rw, z, z) – S(Rw, w, w) , a \|$

Since λ ε [0,1], which gives the desired inequality.

(1.2.2) LEMMA : Let (E, ‖ . , . ‖) be a reflexive 2 – Banach space possessing a weakly sequentially continuous duality mapping J : E $\rightarrow$ E * and $\phi \neq A \subset E$ closed, bounded and convex. Suppose S : A x A x A $\rightarrow$ A is such that

(i) $\| S (y_1 , z_1 , x) – S(y_2, z_2, x) , a \| \leq \lambda [\| y_1 , z_2, a \| + \| z_1 – z_2 ,a \|]$
for all x , y_1 , y_2, z_1 and z_2 ε A and some fixed ε (0,1)

(ii) S (x, x, .) is compact for each x ε A . Then there exists an x ε A such that S (x, x, x) = x.

Proof : Let R : A $\rightarrow$ A be the mapping given by pert (a) of lemma (1. 1. 3). Since S (x, x, .) is continuous for each x ε A by (ii). It follows from part (b) of lemma (1. 1. 3) that R is continuous too, thus it remains to show that R (A) is relatively compact.

For this fix (x_n) ε A^N and set y_n = R (x_n) for all n ε N. Since E is reflexive, A is weakly compact and thus (y_n) possesses some subsequence (y_{k_n}) which converges weakly to a point y ε A. Additionally by (ii) there exists a subsequence $(x_{k_{p_n}})$ of (x_{k_n}) and a point z ε E such that lim S(y, x_{kp_n} , a) = z. Since J is weakly sequentially continuous, we have

$$\lim_{n\to\infty} J\,(y_{\mu_n} - y, a)\ (S\,(y,\ x_{\mu_n}, a) - y) = J\,(o)\,(z - y) = O \qquad -(1)$$

Additionally, since y_n = S (y_n, x_n, a),

$$\begin{aligned} \| y_n - y, a \|^2 &= J\,(y_n - y, a)\,(S\,(y_n, x_n, a) - y) \\ &= J\,(y_n - y, a)\,(S\,(y, x_n, a) - y) \\ &\quad + J\,(y_n, -y, a)\,(S\,(y_n, x_n, a) - S\,(y, x_n, a)) \\ &\le J\,(y_n - y, a)\,(S\,(y, x_n, a) - y) + \| y_n - y, a \|^2 , \end{aligned}$$

for all n ε N . Since ε [0, 1] , this implies that for all n ε N.

$$\| y_n - y, a \|^2 \le \left(\frac{1}{1-\lambda}\right) J\,(y_n - y, a)\,(S(y, x_n, a) - y) \qquad \text{------}\ (2)$$

It follows from (1) and (2) that lim $\| y_n - y , a \|$ = O.

Thus R(A) is relatively compact.

(1 . 2. 5) <u>THEOREM</u> : Let (E , ‖ . , . ‖) be a reflexive 2 – Banach space possessing a weakly sequentially continuous duality mapping and $\phi \neq A \subset E$ closed and convex. Suppose T : A $\rightarrow$ A is weakly asymptotically semi contractive with data (S , (k_n)) and satisfy the following conditions:

(R) for each ε > o there exists an n_o ε N such that for all n $\ge$ n_o and all z ε A $\| S(. , z , z)^{n+1} (z) - S (. , z , z)^{n} (z), a \| < \varepsilon$

Then,

(a) inf. { ‖ x – Tx , a ‖ : x , a ε A } = O

(b) if { (I – T) (A) is closed, it follows that Fix (T) $\neq$ ϕ i.e. T has a fixed point in A. (Fix (T)) denotes the fixed point set of T.

Proof : without loss of generality we may assume that O ε A. Define $\lambda_n = 1 - \frac{1}{n}$ for all h ε N. Then since A is convex, the mapping S_n given by S_n (x, y, z) = (λ_{n/k_n}) S (. , y, z $)^n$ (x) maps A x A x A in to A, for fixed n ε N. Additionally for fixed x ε A, S_n (x, x, .) is compact and S_n (. , x, x) is a contraction with Lipschitz constant λ_n . This follows immediately from the weak asymptotic semi-contractivity of T. Thus by lemma (1. 2. 4) for each n ε N there exists an x_n ε A S.t.

$$X_n = S_n (x_n , x_n , x_n) = (\frac{\lambda_n}{k_n}) S (. , x_n , x_n)^n (x_n) .$$

Hence, $\| x_n - S (. , x_n, x_n)^n (x_n) \| 1- (\frac{k_n}{\lambda_n}) \|$ diam (A), for all n ε N and so

$$\lim_{n \to \infty} \| x_n - S (. , x_n , x_n)^n (x_n) , a \| = O \qquad \text{-----------} \quad (3)$$

Additionally by (R)

$$\lim_{n \to \infty} \| S (. , x_n , x_n)^n (x_n) - S (. , x_n , x_n)^{n-1} (x_n) , a \| = O \qquad \text{-----} (4)$$

Furthermore, for all n ε N

$$\begin{aligned} \| x_n - Tx_n , a \| &\le \| x_n - S (. , x_n , x_n)^{n-1} (x_n) , a \| \\ &\quad + \| S (. , x_n , x_n)^n (x_n) - S (. , x_n, x_n) (x_n) , a \| \\ &\quad \le \| x_n - S (. , x_n , x_n)^n (x_n) \| + k_1 \| S (. , x_n , x_n)^{n-1} (x_n) - x_n , a \| \\ &\le \| x_n - S (. , x_n, x_n)^n (x_n) , a \| \\ &\quad + k_1 [\| S (. , x_n, x_n)^{n-1} (x_n) - S (. , x_n , x_n)^n (x_n) , a \| \\ &\quad + \| S (. , x_n , x_n ,)^n (x_n) - x_n , a \|] \end{aligned}$$

This together with (3) and (4) implies that lim $\| x_n - Tx_n , a \| = O$ which establishes (a). Claim (b) is a direct consequences of (a) as follows:

For this let (x_n) be a sequence in (I – T) A Such that (I – T) $x_n \to$ (I – T) x. Then (I – T) x ε (I – T) A. Thus form (a) inf (I – T) x = O which implies x = Tx.
Thus Fix(T) $\neq \phi$. //

Now we prove some lemmas before going to our nect result which is an improvement of Theorem (4.2.5).

(1 . 2. 6) LEMMA : Let (E, $\| . , . \|$) be a 2 – nor mea space,
$\phi \neq A \subset E$ and S : A x A x A $\to$ A such that

(i) S (. , x, x) is Lispschitzian with constant for all x ε A and fixed n $\le$O.

(ii) S (x, x. .) is strongly compact for each x ε A. Then for fixed x ε A and fixed m n ε N, the mapping y $\rightarrow$ S (., y, y)^n (x) is strongly compact on A.

Proof : Fix x ε A and define g_n (y) = S (. , y, y)^n (x) for all y ε A and all n ε N. Then for all n $\geq$ 2 and all y, z ε A and every a ε A.

$$\| g_n (y)-g_n (z), a \| \leq \| S (S(., y, y)^{n-1} (x), y, y) - S(S(., z, z)^{n-1} (x), y, y), a \|$$
$$+ \| S(S(., z, z)^{n-1} (x), y, y) - S(S(., z, z)^{n-1} (x), z, z), a \|$$
$$\leq L \| S (., .,y)^{n-1} (x) - S (., .,z)^{n-1} (x), a\|$$
$$+ \| S (S(., ., z)^{n-1} (x), y, y) - S (S (., z, z)^{n-1} (x), z, z), a \|$$
$$\leq L \| g_{n-1} (y) - g_{n-1} (z), a \|$$
$$+ \| S (g_{n-1} (x), y, y)-S (g_{n-1}(z), z, z) \ a \| \quad \text{---------} \quad (5)$$

It follows from (50 by induction that g_n is strongly compact for each n ε N. Indeed g_1 = S (x, x,.) is strongly compact by (ii) and if g_n is strongly compact for some n ε N, then for each sequence (z_j) ε A^N weakly converging to some z ε A, it follows that $\lim_{j\to\infty} g_n$ (x_j) = g_n (z). Additionally by (ii), $\lim_{j\to\infty}$ S (g_n(z), z_j, z) = S_n (g_n (z), z, z). Hence it follows from (50 that $\lim_{j\to\infty} g_{n+1}$ (z_j) = g_{n+1} (z).
Consequently g_{n+1} is strongly compact.

Remark: Clearly each strongly compact mapping T : A → E on a weakly compact subset of A of a 2 – normed space E is also compact. This together with the lemma

(1.2.6) shows that the class of all asymptotically semi-contractive mapping on a weakly compact subset A of a 2 – normed space E is a subclass of the class of all weakly asymptotically semi-contractive mappings on A.

(1.2.7) LEMMA: Let (E, ‖., . ‖) be a reflexive 2 – Banach space and ≠ = A $\subset$E closed, bounded and convex.

Suppose S : A x A x A → A is such that
(i) ‖ S (z_1, y_1, x) – S (z_2, y_2, x), a ‖
$$\leq\lambda [\| z_1 - z_2, a \| + \| y_1 - y_2, a \|]$$

For all x, y_1, y_2, z_1 and z_2 ε A and for each a ε A where λ ε [0, 1].
(ii) S (x, x, .) is strongly compact for each z ε A. Then there exists an x ε A such that (S (x, x, x) = x.

Proof: Let R : A → A be the mapping determined by part (a) of lemma (1.2.3). It follows from (ii) together with part (b) of lemma (1.2.3), that R is strongly compact and thus compact. Hence R has a fixed point x ε A by the Schauder's fixed point theorem, which the desired result because S (Rx, x, x) = Rx.

(1.2.8) THEOREM: Let (E, ‖ .,. ‖ be a reflexive 2 – Banach space and ø ≠ A ⊂E closed bounded and convex. Suppose T : A → A is asymptotically semi-contractive with data $(S(k_n))$ and fulfills the asymptotic regularity condition of theorem (. 1.5). Then

(a) inf { ‖ x – Tx, a ‖: x ε A and for every x ε A } = 0
(b) if (I – T) (A) is closal, it follows that Fix (T) ≠ ø.

Proof: It follows from lemma (. 1.6) that for fixed n ε N and fixed x ε A the mapping y → $A (., y, y)^n (x)$ is strongly compact. Thus the proof of theorem (. 1.5) except the fact that we have to use lemma (.1.7) instead of lemma (.1.4) in order to get a sequence (x_n) ε A^N such that $x_n = (n/k_n) S(., x_n, x_n)^n (x_n)$, for all n ε N. //

Remark: If T is asymptotically semi-contractive than in Theorem (1.2.5), we can drop the assumption that E possesses a weakly sequentially continuous duality mapping as we observe in Theorem (1.2.8).

Now we shall extend a demiclosedness result for asymptotically non-expensive mappings to wider class of asymptotically semi-contractive mappings. But before that we give some definitions and prove some lemmas:

(1.2.9) DEFINITION: Let (E, ‖ .,. ‖) be a 2 – normed space,
Ø ≠A⊂E and (x_n) ε E^N bounded. For each y ε E, set r (x_n),y,a) = lim sup ‖ x_n – y, a ‖

(a) The asymptotic radius of (x_n) with respect to A is defined by R $((x_n)$, A, a) $= \inf_{y \varepsilon A} r((x_n), y, a)$ for each a ε A

(b) The asymptotic centre of (x_n) with respect to A is given by
A ⊂ $((x_n)$, y, a) = {y ε A, r ((x_n), y, a) = R ((x_n), A, a) }, for each a ε A.

(1.2.10) LEMMA : Let (E, ‖.,. ‖ be a 2 – Banach space satisfying opial condition, ø ≠ A⊂ E, (x_n) ε E^N and x ε A such that (x_n) – x. Then A ⊂ $((x_n)$, A, a) = {x}.

Proof: This follows from [21, lemma 3] by simple computation only that for all y, a ε $E/_{\{x\}}$ lim sup ‖ x_n – x , a ‖ < lim sup ‖ x_n – y, a ‖/

(1.2.11) <u>LEMMA</u>: Let (E, ‖.,. ‖) be a uniformly convex 2 – banach space, $\emptyset \neq A \subset E$ closed and convex. $\{x_n\} \varepsilon A^N$ bound and z ε A with $A \subset ((x_n), A, a) = \{z\}$. Suppose $(y_n) \varepsilon A^N$ is such that

$$\lim_{n\to\infty} r((x_i)_{I \varepsilon N}, y_n, a) = R((x_i)_{i \varepsilon N}, A, a)$$

Then lim (y_n) = z.

<u>Proof</u>: This follows easily from [3] hence we omit it.

(1.2.12) <u>THEOREM</u>: Let (E, ‖.,. ‖) be a uniformly convex 2 – Banach space satisfying opial conditions and $\emptyset \neq A \subset E$ closed and convex. Suppose T : A→A is asymptotically semi-contractive with data (S, (k_n)). Then) (I–T) is demi-closed with respect to 0 (this means that for each sequence (x_n) ε A^N and each point x ε A such that $(x_n) \to x$ and lim ‖ x_n ,-Tx_n, a‖=0 it follows that Tx = x.

<u>Proof</u>: Let (x_n) ε A^N and x ε A be such that $x_n \to x$ and lim ‖ $x_n - Tx_n$, a ‖ = 0.

For all n, m ε N, we have

$$\| x_n - S(., x, x)^m (x), a \| \leq \| x_n - S(., x_n, x_n)(x_n), a \| + \sum_{\gamma=2}^{m} \| S(., x_n, x_n)^{\gamma-1}(x_n) - S(., x_n, x_n)^{\gamma}(x_n), a \| + \| S(., x_n, x_n)^m (x_n) - S(., x, x)^m (x), a \|$$

$$\leq \| x_n - S(x_n, x_n, x_n), a \| + \| x_n, - S(x_n, x_n, x_n), a \| \sum_{\gamma=2}^{m} k_{-1} + \| S(., x_n, x_n)^m (x_n) - S(., x_n, x_n)^m (x), a \| + \| S(., x_n, x_n)^m (x) - S(., x, x)^m (x), a \|$$

$$< (1 + \sum_{=1}^{m-1} k) \; \| x_n - T x_n, a \| + k_m \| x_n - x, a \| + \| S(., x_n, x_n)^m (x) - S(., x, x)^m (x), a \|.$$

It, follows from lemma (. . 6) that $y \to S(., y, y)^m (x)$ is strongly compact which in turn implies that $\lim_{n\to\infty} S(., x_n, x_n)^m x) = S(., x, x)^m (x)$, taking into account that $x_n - x$. Thus it follows from the inequality above that, for all m ε N.

$\lim_{n\to\infty}$ sup. ‖ x_n – S. (., x, x)^m (x), a ‖ < k_m $\lim_{n\to\infty}$ sup ‖ x_n – x, a ‖ hence, r ((x_n), S (., x, x)^m (x), a) < k_m r ((x_n), x, a)

Since A ⊂ ((x_n), A, a) = {x} by lemma (. . 9), this leads to R ((x_n), A, a) < r ((x_n), S (., x, x)^m (x)), a) < k_m R ((x_n), A, a) where $\lim_{m \to \infty} (k_n) = 1$. This

$\lim_{m \to \infty} r((x_n), S(., x, x)^m (x), a) = R((x_n), A, a)$, from which it follows by lemma (. . 10) that $\lim_{m \to \infty} S(., x, x)^m (x) = x$. Since S (., x, x) is continuous this in turn implies that S (x,x,x) = x hence Tx = x, //

Now we are in position to prove our main result concerning the existence of fixed point of asymptotically semi-contractive mappings.

(1.2.13) <u>THEOREM</u>: Let (E, ‖.,. ‖) be a uniformly convex 2 – Banach space satisfying opial's condition and $\emptyset \neq A \subset E$ closed, bounded and convex. Suppose $T: A \to A$ is asymptotically semi-contractive and fulfills the asymptotic regularity condition (R) of Theorem (. .5). Then there exists an $x \in A$ such that Tx = x.

<u>Proof</u>: Since every uniformly convex 2 – Banach space is reflexive, it follows from theorem (. . B) that there exists a sequence $(x_n) \in A^N$ such that $\lim \| x_n - Tx_n, a \| = 0$. Furthermore, as a consequence of a weak compactness of A, there exists some sub-sequence (x_{k_n}) of x_n which converges weakly to a point $x \in A$. Since I-T is demiclosed with respect to 0 by theorem (. .11) it follows that x is a fixed point of T. //.

\ **REFERENCES**

\(1). Browder, F.E.: Semi-contractive and semi accer-ative mappings in Banach spaces.

Bull. Amer. Math. Soc. 74 (1968) 660-665.

(2). Schu, J.: Fixed points of Mappings satisfying semi-contractivity conditions. J.Austral. Math. Soc. (Series A) 53 (1992) 25-38

(3). Nageswara Rao, B: Fixed Point Theorems through Rational Expression International eJournal of Mathematics and Engineering Vol.2:issue.2,166 (2012) 1555 - 1581

(4).Nageswara Rao, B: Fixed Point Through Contractive and Pseudo Contractive Mappings; International eJournal of Mathematics and Engineering. Vol.2:issue.2168 (2012) 1588 – 1597

(5).Nageswara Rao, B: Fixed Point Theorems for Weak Commutating Mapping; International Journal of Mathematical Sciences,Technology and Humanities Vol.2:issue.2, 42 (2012) 422 – 439

(6).Nageswara Rao, B: SOME FIXED POINT THEOREMS IN 2 – METRIC SPACES: International eJournal ofMathematics and Engineering Vol.2:issue.2,164(2012) 1522 – 1545