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Temperature Distributions and Heat Transfer Rates during Pre-cooling of Spherical Bodies

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Abstract: Here we have considered the problem of determination of temperature distribution and heat transfer rate during hydro cooling, a precooling method, of tomatoes as spherical products. Temperature values measured at the center and mass-average of the products were converted to the dimensionless form to obtain the dimensionless temperature distributions and variations of heat transfer rate. We have computed the dimensionless temperature values and heat transfer rates using appropriate boundary and initial conditions

Keywords: Heating transfer rate, precooling, forced convection, Biot number, Fourier number

AMS Mathematics subject classification: 34B07, 34B40

1. Introduction

Precooling is a cooling process which includes the removal of the stored heat energy from harvested food products in the shortest time to prevent their spoilage and to maintain their quality. For practical applications, accurate prediction of the cooling rates, thermal properties, temperature distributions and heat transfer rates in the precooling of solid products of various shapes is very much essential.

Most of the research works carried out on precooling are based on the studies of hydro cooling, air cooling, vacuum cooling and on hydro air cooling methods. Many studies have been focused on the heat and mass transfer analysis during forced – air precooling. Pflug and Blaisdell [6], pflug and kopelman[7] have given an extensive review on the methods of analysis of precooling. Ranade and Narayan Khedkar[8] have conducted a study by using the spray type hydro cooling system to determine thermal characteristics and temperature changes of some fruits and vegetables. Dincer[4] has also determined the temperature distributions of cylindrical products for quenching experiments in the water. Ansari, Charan and Verma[2], Bhowmick, Hayakawa[3], Anasri and Afaq[1] and others have done considerable work. Kakac and Yener[5] have described that a great variety of engineering problems involve the transient heat transfer which can be divided into two groups, periodic and non-pariodic. Precooling of spherical products placed in a cold water bath will come under non-periodic type. In this case, heat is rapidly removed by forced convaction. The temperature gradient during the precooling of spherical products in a cold water bath is a function of the product properties, surface heat transfer parameters, time and distance.

Here we have considered the problem of determination of temperature distribution and heat transfer rate during hydro cooling, a precooling method, of tomatoes as spherical products. Temperature values measured at the center and mass-average of the products were converted to the dimensionless form to obtain the dimensionless temperature distributions and variations of heat transfer rate. We have computed the dimensionless temperature values and heat transfer rates using appropriate boundary and initial conditions for the values of Biot Numbers lies between 0 and 100. In this case of precooling, the transient heat transfer process is influenced by both the internal resistance and the surface resistance because of the high value of the Biot Number.

2. Formulation of the Problem

Consider the spherical product initially at T_i and with its surface $r=R$ immersed in a cold water bath at T_a with the constant convective heat transfer coefficient at the surface. The temperature distribution inside the spherical product is determined as a function of time t and radius R. We have solved the above problem of the transient heat transfer for spherical product.
The governing differential one-dimensional heat conduction equation in spherical coordinates to determine the temperature T(r,t) is,

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r})\right]=\frac{1}{a}\frac{\partial T}{\partial t} \tag{1}$$

Above equation interms of the excess temperature $\phi = T - T_a$ is,

$$\frac{\partial \phi}{\partial t}=\frac{a}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial \phi}{\partial r}) \tag{2}$$

Where ‘a’ is thermal diffusivity.

The initial and boundary conditions are

$$\phi(r,o)=\phi_i=T_i-T_a \tag{3}$$

$$\frac{\partial \phi(o,t)}{\partial r}=o \tag{4}$$

$$-K\left[\frac{\partial \phi(R,t)}{\partial r}\right]=h\phi(R,t) \tag{5}$$

Where K is thermal conductivity.

3. Solution of the Problem

Solving by using the method of separation of variables, we get the transient temperature distribution of the solid sphere as,

$$\theta = \frac{T(r,t) - T_a}{T_i - T_a}$$
$$= \sum_{n=1}^{\infty} \frac{2(Bi.Sin\mu_n)}{(\mu_n - Sin\mu_n.Cos\mu_n)} \qquad (6)$$

$$.\exp(-\mu_n^2 F_O).\frac{Sin(\mu_n.\Gamma)}{\mu_n.\Gamma} \qquad (7)$$

Where,

$$\Gamma = r / R \qquad (8)$$

and dimensionless Biot (Bi) and Fourier (F_o) numbers are :

$$\text{Bi} = \text{h.R/K} \qquad (9)$$

$$F_0 = a.t/R^2 \qquad (10)$$

The Biot number compares the relative magnitudes of surface convection and internal conduction resistances to heat transfer. The Fourier number also compares, a characteristic body dimension with an approximate temperature – wave penetration depth for a given time, t.

The characteristic values of μ_n are the roots of transcendental equation (8). Equation (8) has an infinite number of solutions at a given value of the Biot number. From equation (6), it is clear that the dimensionless temperature distribution is a function of the Bio number (Bi), the Fourier number (F_o) and the dimensionless position ratio as

$$\theta = f(Bi, F_O, \Gamma) \qquad (11)$$

The transient temperature distribution for the center of spherical product is given by,

$$\theta_c = \frac{T(o,t) - T_a}{T_i - T_a} = \sum_{n=1}^{\infty} \frac{2(Bi.Sin\mu_n)}{(\mu_n - Sin\mu_n.Cos\mu_n)}$$

$$.Exp(-\mu_n^2 F_o) \qquad (12)$$

In order to introduce the heat transfer rate distribution for the time interval (o, t), the ratio between the total heat transfer Q from the spherical body and its internal energy Q_i in dimensionless form for $\Gamma = 0.75$ is

$$\phi = \frac{Q}{Q_i} = \sum_{n=1}^{\infty} \frac{6(Bi.Sin\mu_n)^2}{\mu_n^3(\mu_n - Sin\mu_n.Cos\mu_n)} \left[1 - \exp(-\mu_n^2 F_O)\right] \tag{13}$$

Where,

$$Q_i = P.V.C_p\,(T_i - T_a) \tag{14}$$

Where P is product density, V is kinematic viscosity and C_p is specific heat.

4. Results and Discussion

The calculation of the convective heat transfer coefficient from the solid products to water is required for the determination of temperature distribution of the spherical product. As given in Holeman, the average convective heat transfer from the spherical product to water in forced convection over a wide range of Reynolds numbers is calculated from

$$(h.D/K_{wt}).\bar{\Pr}^{0.3}(\mu_w^*/\mu^*)^{0.25} = 1.2 + 0.53.\mathrm{Re}^{0.54} \tag{15}$$

Where all physical properties are evaluated at the free – stream conditions, except μ_w^* which is evaluated at the surface temperature of the solid sphere. Here Reynold number is

$$\mathrm{Re} = U.D/\nu \tag{16}$$

Here, h is convective heat transfer coefficient, D is diameter, K_{wt} is thermal conductivity of water, Pr is Prandtl number, U is average flow velocity, v is Kinetic Viscosity and μ^* is dynamic viscosity. Thermal conductivity of the spherical products as given in sweat is

$$K = 0.148 + 0.493W \tag{17}$$

Thermal properties of food products depend on the water content. As given in ASHRAE Hand book of Fundamentals, thermal diffusivity of food products is taken as

$$a = 0.088\text{x}10^{-6} + (a_{wt} - 0.088\text{x}10^{-6})w \tag{18}$$

Where a_{wt} is the thermal diffusivity of water at the product temperature and is taken as $a_{wt} = 0.148\text{x}10^{-6}$.

The thermo physical properties of water are taken as,

$$\Pr = 1324,\ K_{wt} = 0.56\ \text{W/mk},\ V_f = 1.7382 \times 10^{-6}\ \text{m}^2/\text{s}$$

$\mu^* = 17.418\text{x}10^{-6}$ kg/ms, $\mu^*_w = 9.663\text{x}10^{-6}$ kg/ms.

Re = 2013.57

The average convective heat transfer coefficient for spherical product having 70 mm diameter was calculated using equation (15) and it is, h = 673 $\text{w/m}^2\text{K}$.

For spherical product containing water content of 94.5 % the thermal conductivity and thermal diffusivity are calculated using equations (17) and (18) and are given by,

K = 0.6138 W/mK

a = $0.1447 \text{ x } 10^{-6} \text{ m}^2/\text{S}$.

With the above values of h, k and for radius R = 0.035 m, the Biot number is calculated and it is,

Bi = 38.4

Since the Biot number (Bi) lies between 0 and 100, we can consider the most realistic case of precooling in which both the internal and surface resistance to the heat transfer in spherical product occurs.

The dimensionless temperature distributions at the center of the sphere θ_c given by equation (12) are evaluated, by taking first 18 values of μ_n with in the limits of F_o from 0 to 0.28 and $\Gamma = r / R = 0$ (at the center) and are presented in Figure 1.

The heat transfer rate values from spherical product to the water medium are evaluated from equation (13) and are shown in Figure 2. Here we had taken temperature of water as 1^oC and flow velocity of water as 0.05 m/s.

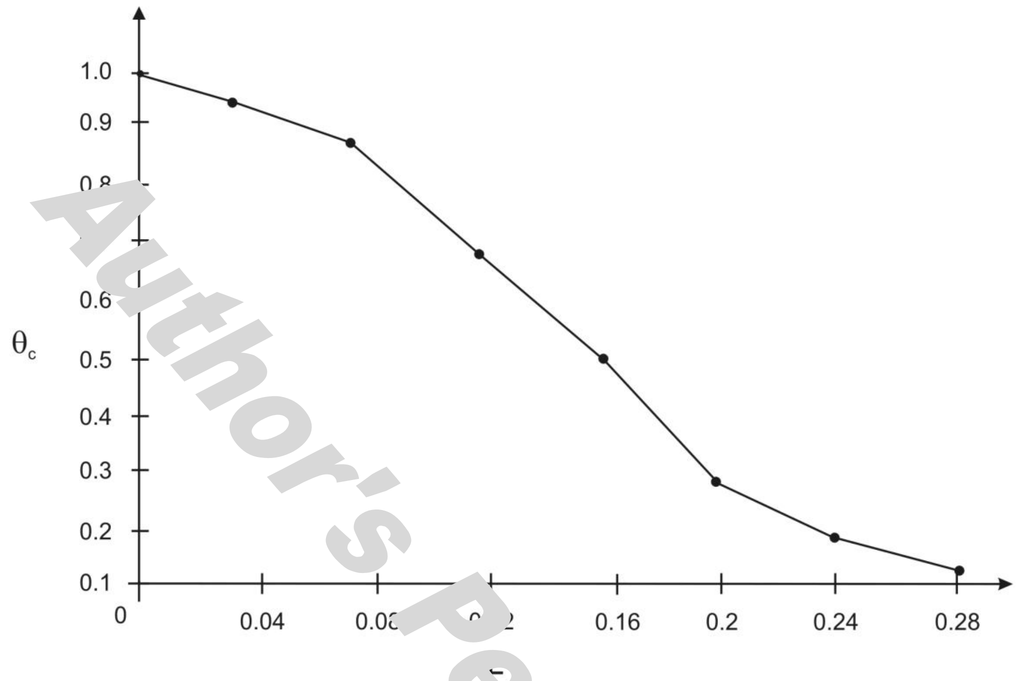

Fig. 1: Dimensionless temperature distributions against the Fourier Number (F_o) for the center case.

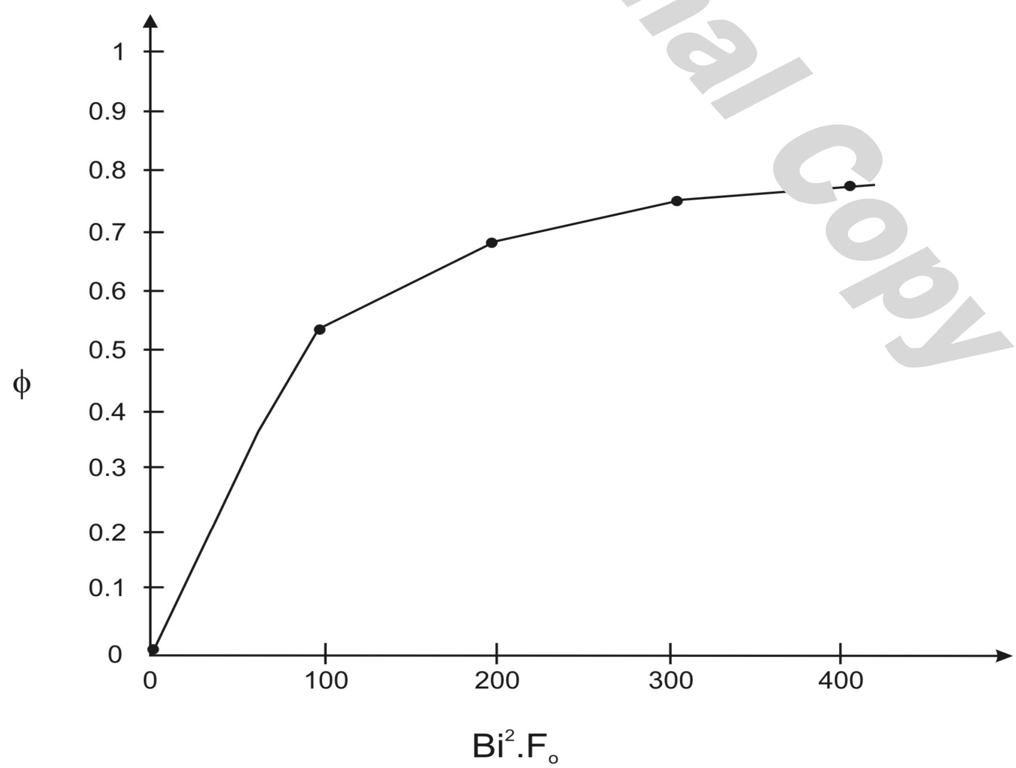

Fig. 2 : Heat transfer rate variations

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